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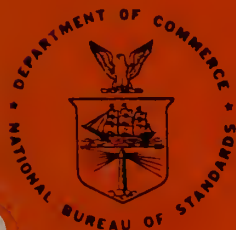
# Mathematical Modeling of Fires

R. S. Levine

Center for Fire Research  
National Engineering Laboratory  
National Bureau of Standards  
U.S. Department of Commerce  
Washington, DC 20234

September 1980

Final Report



U.S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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**U.S. DEPARTMENT OF COMMERCE, Philip M. Klutznick, *Secretary***

**Luther H. Hodges, Jr., *Deputy Secretary***

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# MATHEMATICAL MODELING OF FIRES

R. S. Levine

## Abstract

This presentation has three technical parts, and ends with audience participation and recommendations. First, a brief discussion of fire growth in a compartment is presented, showing why we need full scale tests, or a mathematical model adequately simulating such growth. The second part of the talk describes what several Federal agencies and their grantees are doing to bring about the necessary engineering and mathematical capability for this modeling. The third part illustrates some problems that may be of interest to fire protection engineers that can be solved relatively simply by using fragments of the modeling capability now available.

Then a discussion was held with the audience to determine modeling needs. Should we provide a series of simple models, each applicable to a limited range of problems, or a major comprehensive model, accessible from a computer terminal, that will solve a very wide range of problems? The audience decided both were needed.

Key words: Fire; fire engineering; fire safety; mathematical modeling; modeling application.

## 1. PART I.

### Modeling and the Compartment Fire.

Figure 1 [1]<sup>1</sup> is an illustration of the processes occurring in a fire in a compartment with an opening in it. The fire over the burning object generates a plume of hot gas that entrains air,  $\dot{M}_i$ , from the lower layer, and adds a flux of hot, partly unburned gas,  $\dot{M}_p$ , to the hot ceiling layer. Early in the fire, before the ceiling layer has grown below the doorway height,  $H_i$ , unburned air flows out the doorway to make room for the hot, lower density gas in the ceiling layer. Later, for a short time, both hot ceiling layer gas and unburned air flow out the doorway; then as the ceiling layer approaches the thickness  $h_L$ , ceiling layer gas flows out and outside air flows in. At the neutral axis, the pressure outside the room and inside are equal. Buoyancy forces cause the pressure above the neutral axis inside the room to be greater than the outside pressure, and lower than the outside pressure below the neutral axis.

The flow of the ceiling layer to the exterior is of key concern to the safety of the rest of the structure, since this is the source of smoke and toxic gases. The other rooms in the structure are generally made untenable by smoke obscuration or toxicity before they are untenable due to heat [2].

---

<sup>1</sup>Numbers in brackets refer to literature references listed at the end of this paper.

As figure 1 indicates, the processes within the room react on each other. Thermal radiation from the fire and the hot ceiling layer, and the upper walls and ceiling affect the burning rate (of the outside surfaces) [3] of the burning object, and also heat up other objects in the room, shown here as a "target", until they may eventually ignite. If the flame is spreading, the rate of flame spread, as well as the rate of burning of already ignited surfaces will be affected by the preheating due to this radiation [4].

The plume above the fire and its entrainment of lower layer air is, of course, affected by the burning rate of the fire, which in turn is affected by the thermal radiation, the vitiation of the oxygen content of the lower layer air caused by mixing between the two layers (not shown in figure 1), and drafts due to  $M_1$ . The upper layer gases are cooled by convective heat transfer to the ceiling and upper walls, and this cooling can have a significant influence on the temperature of the upper layer, hence its radiation, and hence the growth rate of the fire.

These interactive effects cannot all be scaled simultaneously in scale models. Particularly thermal radiation, which depends on the optical path length through the hot gases, and the plume entrainment, which depends on the size of the plume, its height, and combustion in it, are difficult to scale.

So the Fire Protection Engineering Fraternity is rightly skeptical of small scale tests, and has confidence only in realistic full scale tests. These, however, are expensive and difficult to carry out. Reliable, validated mathematical models would be valuable either to extend the results of full scale tests to see the effect of desired changes or to avoid the necessity of the test in the first place. Since the mathematical model must reproduce the interactions described above, where each process is affected by the other processes, it consists of a set of mathematical equations that must be solved simultaneously, and usually iteratively, and is only practically done on a computer.

There are two kinds of fire compartment models being developed today. The most useful currently are "control volume" models. In these the room and its contents are divided into lumped thermodynamic control volumes, with heat, mass, and momentum balance equations written for each. In figure 1, control volumes are: the burning object, the plume above it (up to the upper layer), the upper layer, the lower layer, the heated walls and ceiling, and the heated surface of the target. Of course, as these are further subdivided into control volumes, the program becomes more versatile and more complicated.

As previously mentioned, in practice these calculations must be done by computer. I shall illustrate the process with some slides from the Harvard Computer Fire Code II [5]. Two of the subprograms that are used with the overall major program are shown in figures 2 and 3. The main



program, figure 2, records the input data describing the room and the objects in it, physical constants, etc., and then, for a particular time,  $t$ , calculates the various flows, burning rates, radiation fluxes, etc., based on values extrapolated from the previous time step. Then it adjusts these values by an iterative (loop) process until they all balance properly (converge) for the time step. Then it moves ahead one time step and repeats the process. Figure 3 shows one of these sub-programs which calculates the flow through the opening, which in turn calls whichever calculational method is needed for the state of the ceiling layer at that time.

The newer computer model [6] bears only an evolutionary resemblance to what I have just described. It is designed to make the convergence process more efficient in respect to computer time, so it scales all variables to within  $-1$  and  $+1$ , does the mathematics, and rescales them to real values. It is hoped that to the user the architecture of the program will be of little concern. He should be able to use the program by typing English words into a terminal.

For the control volume modeling to be accurate requires the wise selection of control volumes. If the air and ceiling layer flows are complex, involving mixing and recirculation in unpredictable ways, "Field Modeling" may be used. The compartment is divided into a rectangular grid, and the conservation equations are written for each grid cell. The resulting set of equations together with boundary conditions is solved, as a function of time, on a computer and the mathematics predicts the flow field as it varies with time. Combustion and radiation can be approximated, but the approximations required to obtain solutions in a reasonable amount of computer time are of doubtful validity. Three-dimensional effects can be calculated in only a few specialized cases with today's computer capability.

Flow from room to room (figure 4) is only beginning to be attacked [7,8]. The problem is not simple since varying amounts of entrainment and mixing between the ceiling layer and the lower layer occur, especially where the flow has to change direction, such as at a doorway. Also, the ceiling flow is cooled by heat transfer, both convective and radiative. The ceiling layer in the second room may well have substantially greater mass flow, but be cooler and more dilute in smoke and toxic gases than the flow leaving the room of origin.

## 2. PART II.

### The Agency Role.

Several agencies are concerned with developing mathematical modeling capability for their responsibilities in fire safety. The Japanese Building Research Institute took an early lead in this [9,10]. They paid particular attention to the radiative ignition and spread of fire on walls and other surfaces, which reflects their concern with this problem in Japanese housing. Figure 5, from reference [9], shows this fire spread concept.

In this country, Professor Emmons and his co-workers at Harvard University [1,11] have generated the most comprehensive computer program. This work has been done in close collaboration with Factory Mutual Research Corporation (see for instance reference [12]), and is continuing under a research grant from the National Bureau of Standards (NBS).

NBS is carrying out other work, both in-house and by grant. Dr. James Quintiere [3,13] has developed a series of quasi-steady state models, one of which will be used in part III of this paper. Research on the various processes important to these models is carried out both in-house and at universities and other research institutions funded by grants from the NBS Center for Fire Research. These processes include plume combustion and entrainment, thermal radiation from soot, convective heat transfer in the ceiling layer, flame spread and ignition as affected by thermal radiation, flow through openings and from room to room, and smoke toxicity. As better information from this research becomes available, it will be incorporated in new versions of the computer programs.

The Federal Aviation Administration (FAA) and the National Aeronautics and Space Administration (NASA) are cooperating in a comprehensive program to use modeling to achieve aircraft fire safety. Both the effects of a fuel pool fire outside a crashed aircraft and fire within the cabin are subjects of mathematical modeling efforts. The major aircraft control volume modeling effort (figures 6 and 7) is being carried out by the University of Dayton Research Institute [14], with confirmatory fuselage fire testing done in-house by the FAA, by NASA, and their grantees. Field equation modeling is being applied to the pool fire problem and to the flow of gases down the long narrow fuselage (a mixed ceiling layer cannot be assumed). The Naval Research Laboratory is similarly concerned with modeling compartment fires.

Other work is being done at the Illinois Institute of Technology Research Institute [15,16] with special reference to the burning of furnishings in a compartment prior to flashover.

Obviously it is advantageous to all concerned to provide a vehicle for these various contributors to cooperate and benefit from each other's efforts. The Ad Hoc Group on Mathematical Fire Modeling has therefore been formed. Its composition is shown in figure 8. The Group is divided into three subcommittees.

The Synthesis, Models and Scenarios Committee is chaired by Professor Howard Emmons of Harvard University and is in turn divided into two subcommittees. The subcommittee on User's Needs is chaired by Mr. Irwin Benjamin. Its duties are to impact the development of the final program or programs so that they will be of maximum benefit to the users. This can be accomplished by advising the modelers what to calculate, and later to facilitate user adoption of the validated programs. The



subcommittee on Programs is chaired by Dr. James Quintiere of NBS. Its goal is to arrive at a program, or set of programs, that fit the most important fire scenarios.

The Committee on subprograms, under Dr. John deRis of Factory Mutual Research Corporation, is concerned with developing and validating subprograms, such as models of ignition, flame spread, air and product gas circulation, etc. These subprograms will be improved by using the results of research programs at various universities, government laboratories, and other organizations, when cast in mathematical form.

The Definition and Coding Committee, under Dr. John Rockett of NBS, selects standard computer nomenclature and standard formats for both the program and the subprograms, and for full scale testing. The latter will make it possible for all of the investigators to use data from full scale tests carried out at various institutions.

I am Chairman of the Steering Committee, which consists of representatives of the funding agencies and the three Chairmen. The funding agencies will cooperate as best they can to facilitate the most important portions of the work.

Benefits expected from the validated programs are listed below and in figure 9 in the order of expected fulfillment.

The first benefit is to permit the results of full scale tests to be extended to other conditions, resulting in an increased body of knowledge of the importance of various parameters, especially early in the fire.

The second benefit is to allow us to direct our research resources to the most important research areas. Sensitivity analyses of the programs show which subprograms are really important, rather than merely technically interesting.

The third benefit will be to allow us to develop meaningful fire safety property tests. Those now in use are generally based on the intuition of practitioners, and their applicability is sometimes in doubt.

The fourth, and major benefit, is to provide a new quantitative tool for the development of design criteria applicable to fire safety in rooms, room-corridor combinations, and buildings, in both early and late stages of fire.

### 3. PART III.

This portion of the paper is intended to illustrate how some of the information developed to date in the mathematical modeling effort can be used, using nothing more complicated than algebra, to solve fire protection problems that would have been nearly impossible a few years ago. Presented as appendices to this paper are samples of the application of computer modeling to solve two problems.

Appendix A is an attempt to calculate the upper layer depth in a closed room (leak under the doorway) as a function of the amount of material burned. The actual complete calculation would have been quite complex, since the upper layer is formed by both the combustion products and the air they entrain in the plume. The entrainment, in turn, depends on the height of the plume between the burning material and the bottom of the upper layer. The correlations developed by Professor Edward Zukoski (on a grant from NBS) were used, and the calculation using his work is straightforward. Prof. Zukoski's publication on this part of his work is appended as Appendix B. In Appendix A, the gas temperature was calculated assuming there was no heat transfer to the walls of the room, and that the leak was in the lowest part of the room. Other assumptions are possible using Appendix B.

Appendix C is a rough estimation of the contribution of a "target" material to the toxicity of the gases leaving a room. The target material is heated by radiation from the hot gases in the ceiling layer. Its pyrolysis gases are assumed to be substantially more toxic than the combustion products of the room fire, but it doesn't decompose until it reaches a relatively high temperature. By that time, the gas flow out of the room from the primary fire is quite large, and only if the exposed area of the target material were large enough to create, say, 1/1000 of this flow, would the target material be a factor in fire safety. The pyrolysis rate of the target material as a function of its temperature is not included in Appendix C, but provision is made for it. This would, of course, require a separate laboratory experiment on the material.

To solve this problem, Dr. Quintiere's relatively simple quasi-steady state "RUNF" computer program at NBS was used to calculate the upper layer height and flowrate and temperature. To the user this is no more difficult than typing the room dimensions, doorway size, and primary fire heat release rate into a terminal. Without the computer program, a great deal of work would be required to obtain these data, which are the basis of the rest of the calculation.

This paper was prepared for presentation to a Society of Fire Protection Engineers' symposium on Systems Methodologies and Some Applications. The audience was asked whether their goals were best met by relatively simple models like those used in the appendices, or a comprehensive model, requiring access to significant computer capability, that would provide answers directly. The consensus was that these practitioners needed both. A calculation that could be examined in detail, as found using a simpler model, would provide their clients more confidence than a number printed by a computer. On the other hand, the full computer model will provide answers that are more accurate and comprehensive than can be obtained practically by simpler models.

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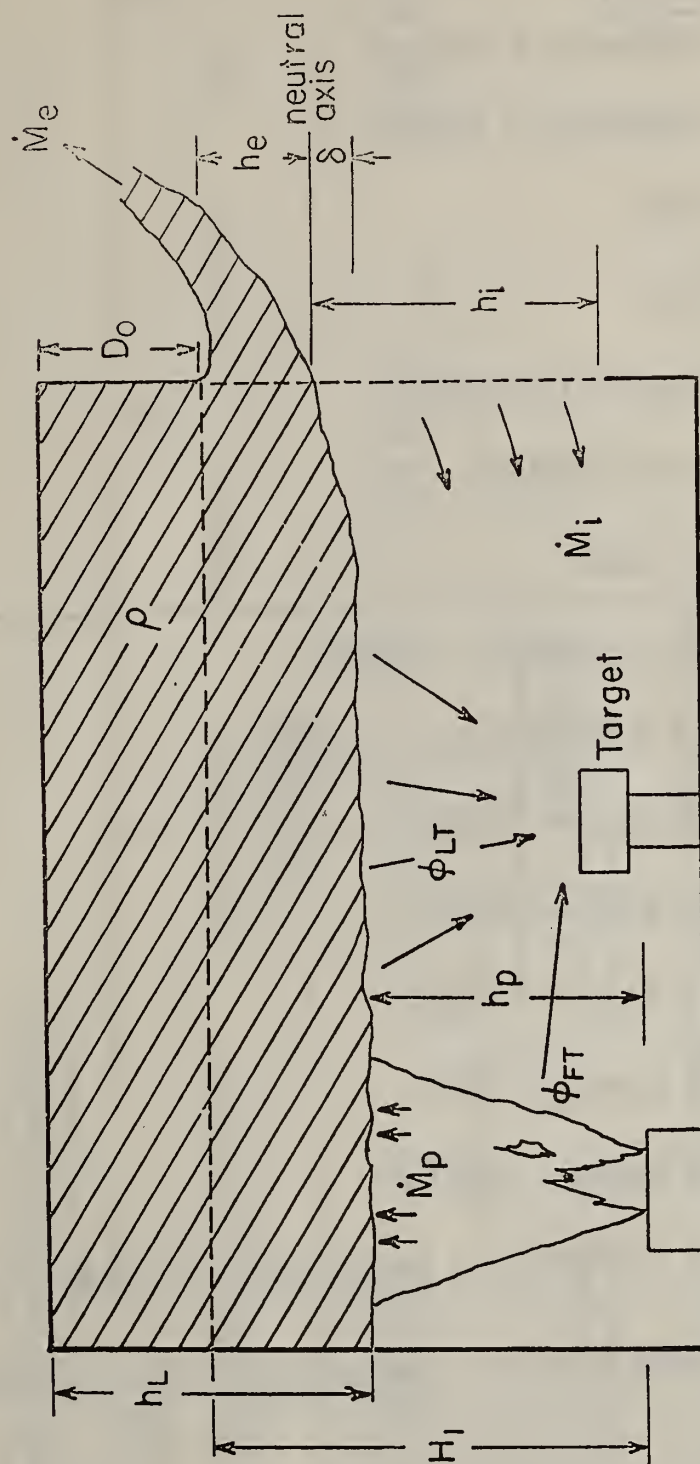


Figure 1. Schematic of the enclosure, showing mass fluxes in and radiative fluxes to the target from layer and flame

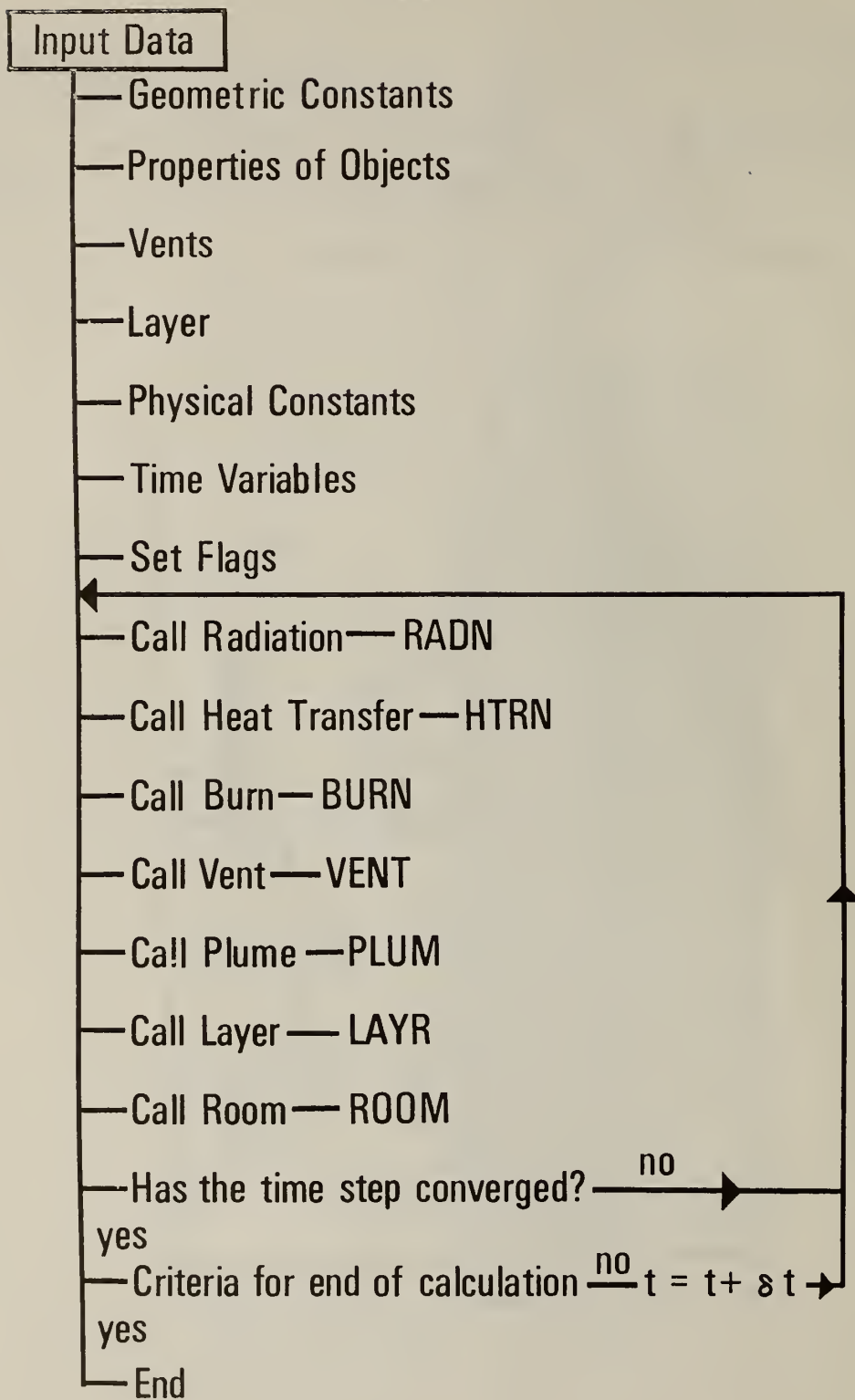


Figure 2. Computer fire code - main program

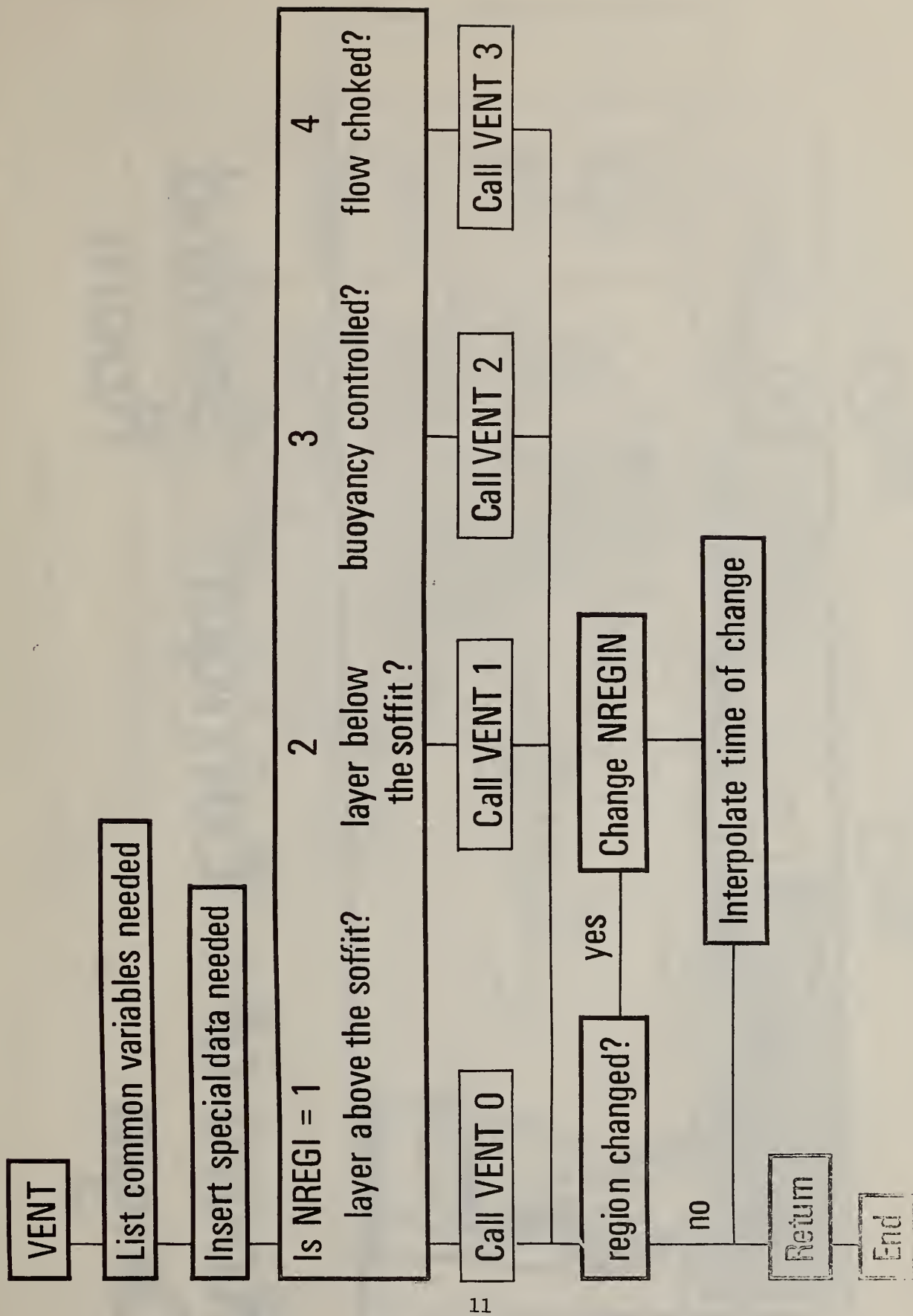
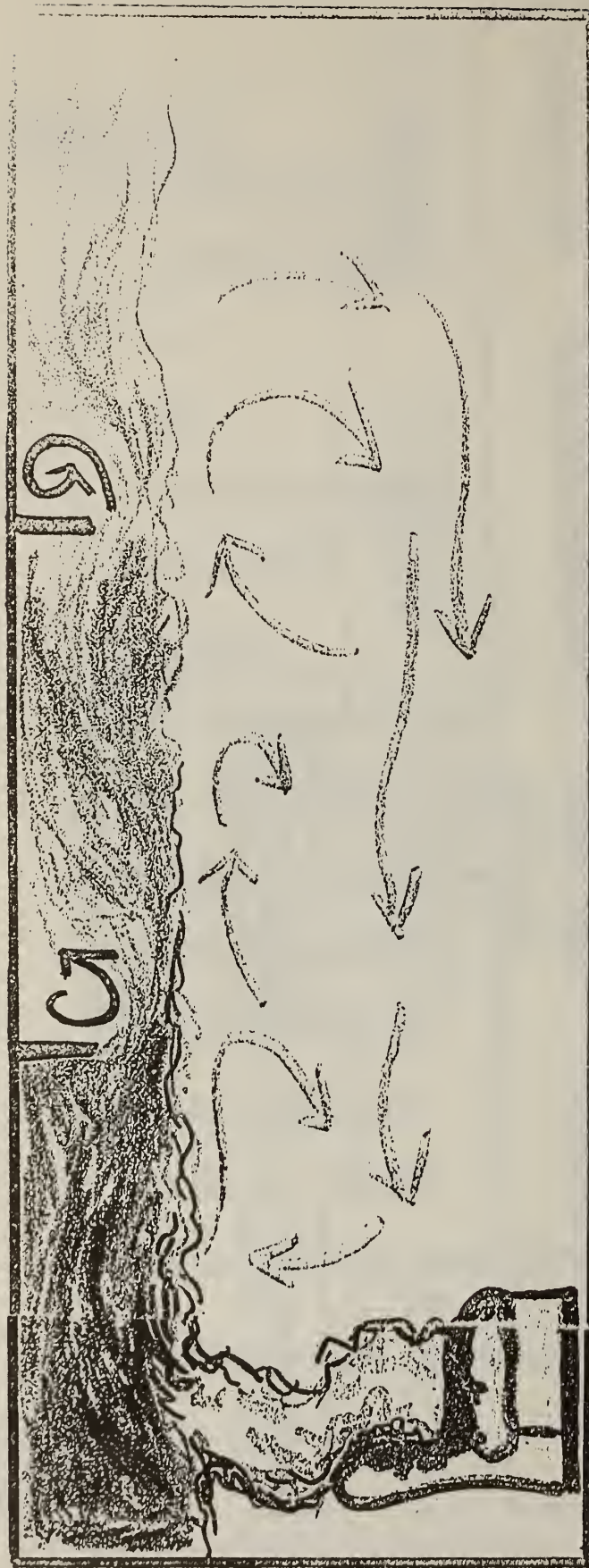


Figure 3. Subroutine vent

Me1 Me2



# Room of Origin      Corridor      Second Room

Figure 4. Path of flame spread



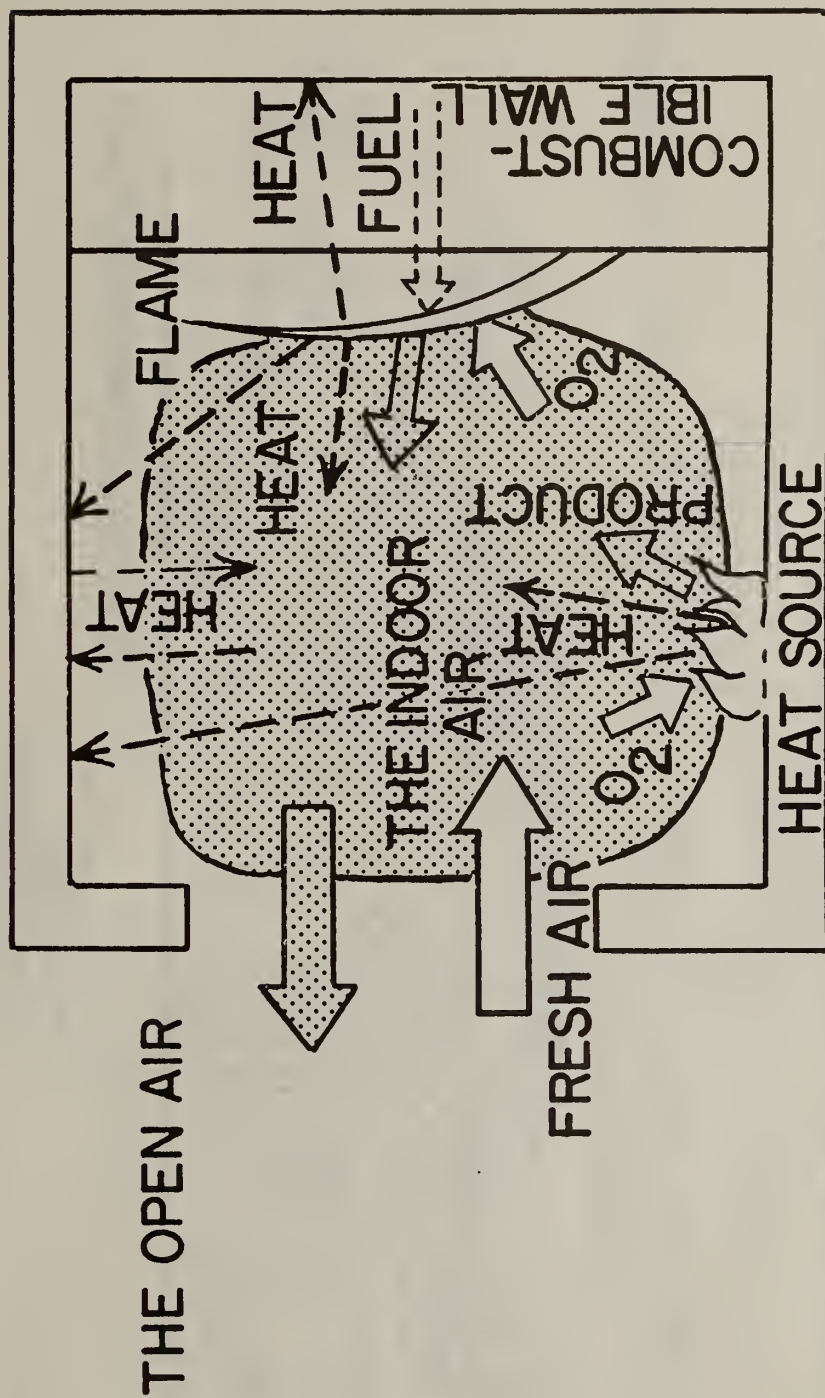


Figure 5. Diagram of a compartment fire

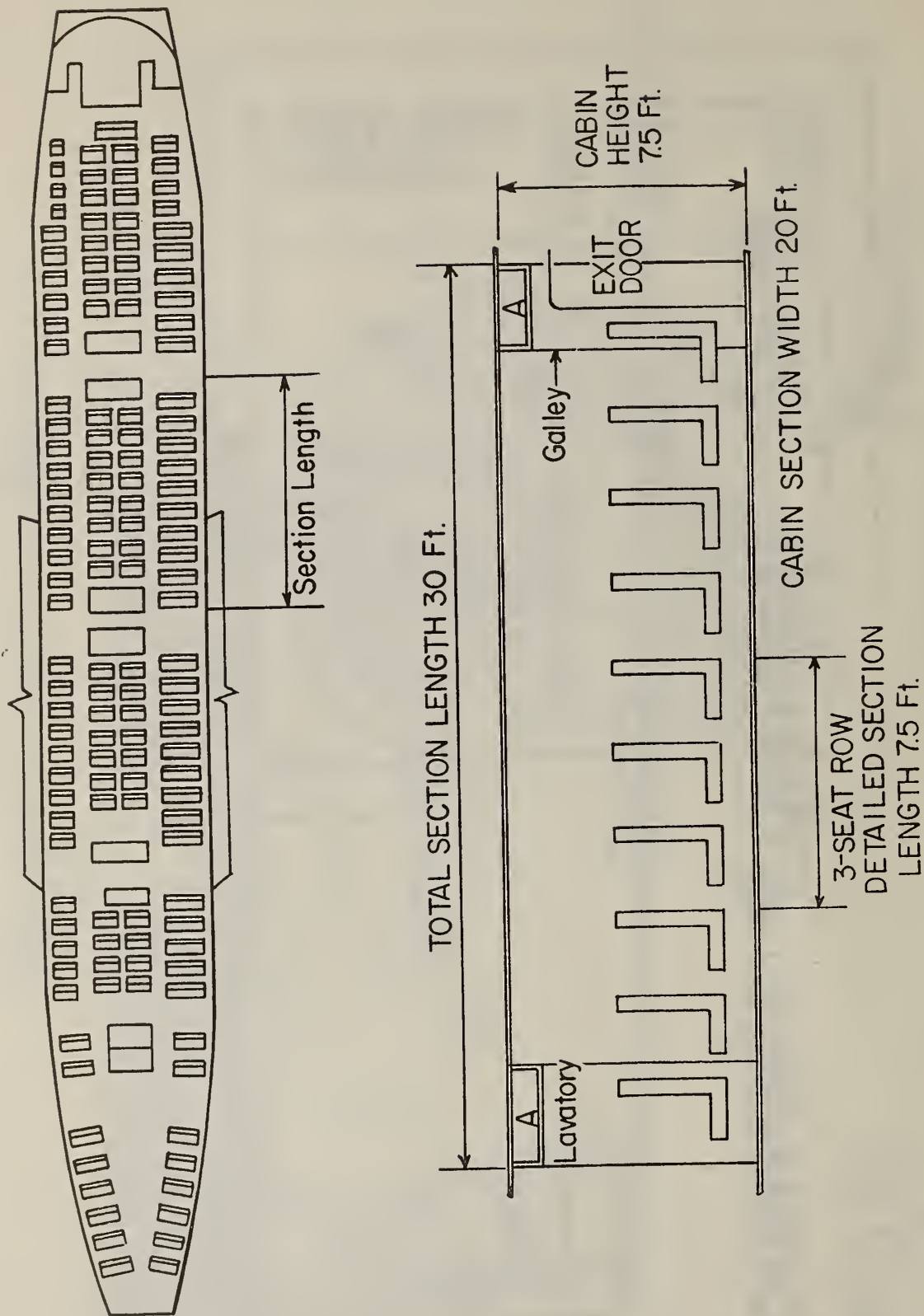
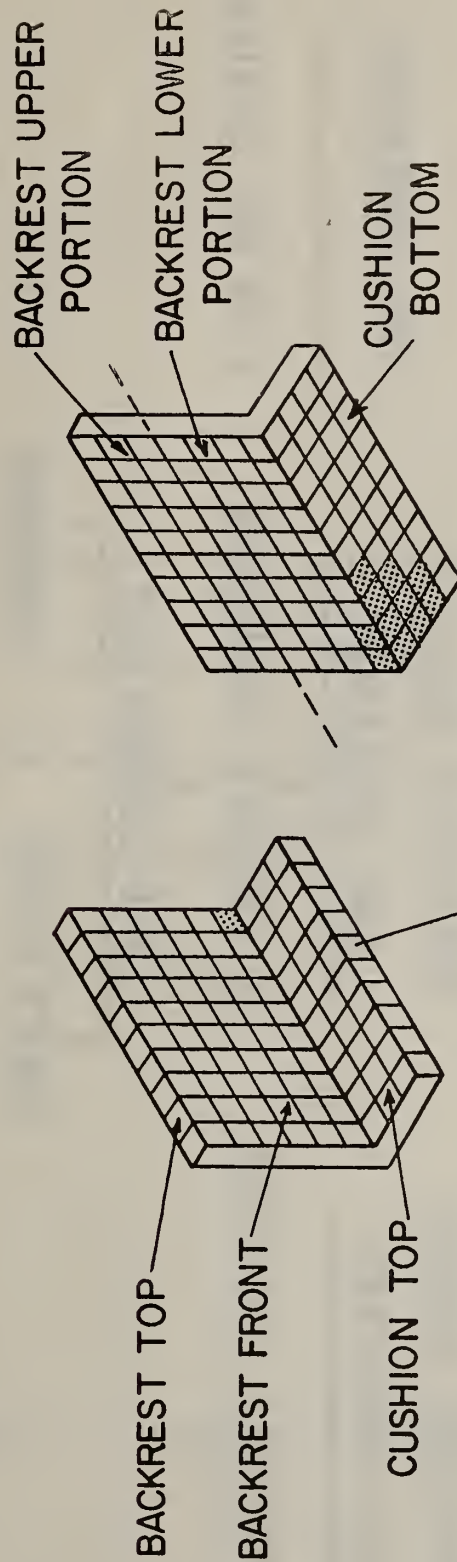


Figure 6. Typical wide-body transport aircraft cabin arrangement



CUSHION FRONT

FRONT AND TOP SEAT SURFACES

BACK AND BOTTOM SEAT SURFACES



FLAMING

Figure 7. Flame spread on seat group 1 at 100 seconds

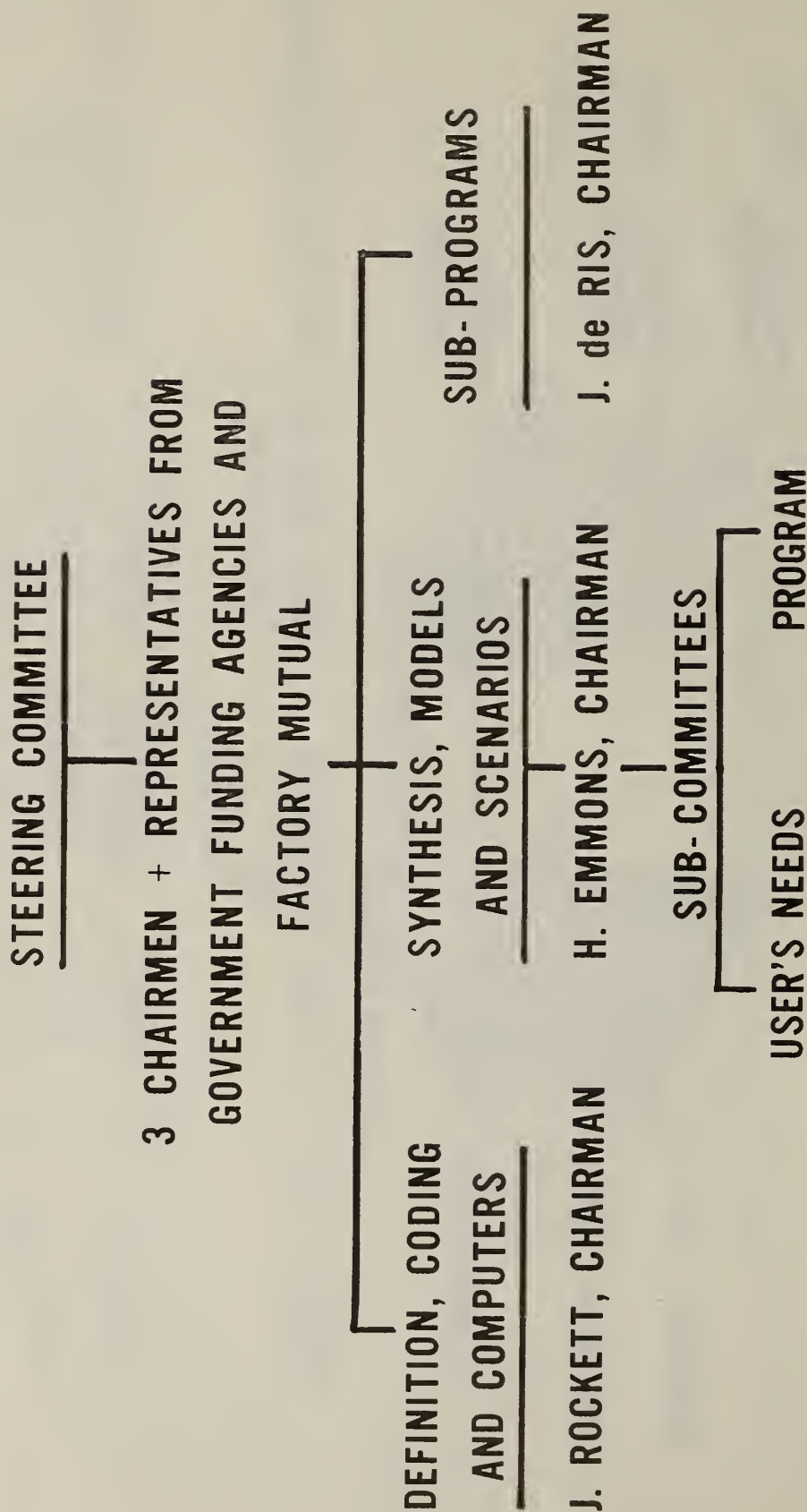


Figure 8. Steering committee organization

- ☐ Extend Full Scale Test Results to Different Conditions
- ☐ Delineate Important Research Areas
- ☐ Define Meaningful Fire Tests
- ☐ Generate Design Data

Figure 9. Benefits - mathematical models of fire





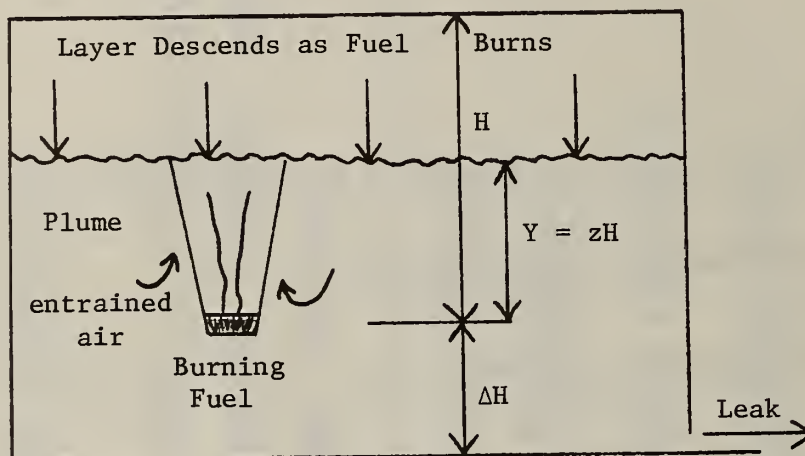
APPENDIX A

MEMORANDUM FOR Those Listed

From: R. S. Levine, Chief  
Fire Science Division

Subject: Math Analysis of the "Closed Room" Toxicity Test

1.0 Problem Statement



The problem is to estimate whether the final toxic gas in a full scale room test will correspond to the gases in a smaller scale apparatus with the same "loading" (weight of original material per  $m^3$  of gas volume). We will calculate only the likely results of the full scale room burn.

So: Find the level of the ceiling layer in a room (door closed, leak under the door) as a function of amount of fuel burned (assume wood) at 10, 20, 30, 40  $gm/m^3$  loading, and the gas composition and temperature of that layer. Room = 10' x 10' x 8' high = 3 x 3 x 2.44 m = 22  $m^3$ . The source is localized, but the products form a ceiling layer that is toxic.

Reference: Zukoski, E. F., "Development of a Stratified Ceiling Layer in the Early Stages of a Closed-Room Fire", Fire & Materials, Vol. 2, No. 2, 1978 (R7800406 in FRIS).

Zukoski gives the results of an analysis of the height of the hot gas layer in a room, where the plume below the layer entrains fresh air. The results of the analysis require only arithmetic to use. He shows (part 7 of above) that the rate of heat addition has only a small effect on the layer level when the same total heat addition has been reached. Therefore, we arbitrarily set a burning rate of 0.1 gm/sec/m<sup>3</sup>. The results will be applicable to any situation where a major part of the thermal energy is not lost as heat transfer to the walls. This latter problem will be calculated in a future memo.

Let us assume the fuel is wood, (C<sub>1.1</sub>H<sub>2</sub>O)<sub>x</sub> burned at 80% combustion efficiency. Heat of combustion is about 5300 cal/gm. Fire size per m<sup>3</sup> is then 5300 cal/sec at a burning rate of 1 gm/sec/m<sup>3</sup> or (5300 cal/sec) (4.187 watts/cal/sec)(22m<sup>3</sup>) = 490 Kw. So burn at 0.1 gm/sec/m<sup>3</sup> = 49 Kw = 2.2 gm/sec = 11,700 cal/sec. (Zukowski did most of his work at about 100 Kw.) A loading of the products of combustion of 10 gms of wood per m<sup>3</sup>, corresponds to burning 22 x 10 = 220 gms of wood (without loss of product).

2.0 In Zukoski's paper, figure 2, plots  $(Q^*)^{1/3} \tau =$  vs  $y$

where  $Q^* = Q/\rho_c C_{\rho} T_c \sqrt{gH} H^2$

$$\tau = t (\sqrt{g/H}) (H^2/S)$$

Height of ceiling layer =  $Y = yH$

$t$  is time, seconds, for layer to descend to  $y$

$H$  is room Height, meters

$S$  is room area, m<sup>2</sup>

$g$  is gravitational constant 9.8 m/sec<sup>2</sup>

Q is heat addition rate from fire, cal/sec

$C_p$  = original (lower layer) room air specific heat

$$= 0.24 \text{ cal/gm}^\circ\text{K}$$

$\rho_c$  = density of original room air ( $1.3 \times 10^3 \text{ gm/m}^3$  at  $273^\circ\text{K}$ )

T = temperature of original room air,  $^\circ\text{K}$

Let the room be  $10' \times 10' \times 8'$  high

$$= 3.05 \times 3.05 \times 2.44 \text{ m}$$

Calculation Method - find where layer is in a time corresponding to the burning time of the fuel at  $2.2 \text{ gm/sec}$

$$Q^* = \frac{2.2 \times 5300 \text{ cal}}{\text{sec}} \frac{\text{m}^3}{1300 \text{ gm}} \frac{\text{gm } ^\circ\text{K}}{0.24 \text{ cal}} \frac{\text{Sec}}{275 ^\circ\text{K}} \frac{4.86 \text{ m}}{4.84 \text{ m}^2}$$

$$\sqrt{gH} = 9.8(2.44) = 4.88 \text{ m/sec}$$

$$Q^* = 2.6 \times 10^{-3}(2.2) = \underline{\underline{0.0058}}$$

$$(Q^*)^{1/3} = 0.18$$

gm fuel	$T_{\text{sec}}$	$\tau$	$Q^{*1/3}\tau$	Y	ceiling layer height = YH
220	100	128	23	0 at $Q^{*1/3}\tau=14$	0 ← gas exits before 220 gm total fuel
440	200	256	40.5		
660	300	385	61	or time = 60 sec.	These discharge some combustion products
880	400	505	80	( $t=14/.18=77.8$ )	through the floor leak.

Layer hits floor at (220 gm) ( $60 \text{ sec}/100 \text{ sec}$ ) = 132 gm of fuel burned.

$$\tau = t (g/M)^{1/2} (H^2/S^2) = t (9.8/2.44)^{1/2} (2.44^2/3.05^2) = t (2.0)(0.64) = 1.28t$$

ceiling layer temperature

$$\text{eq (25)} \quad \rho_h/\rho_c = [(1-(Q^*\tau)/(1-y))] = 1-(0.0058)(77.8) = 1-0.45 = \underline{\underline{0.55}}$$

If the initial temperature is  $20^{\circ}\text{C} = 293\text{K}$ , final is  $1/.55 (293) = 533^{\circ}\text{K} = 260^{\circ}\text{C}$  T final (after 124 gm fuel burned and ceiling layer has reached the floor).

This part of the calculation shows that, if the fuel burns at 80% comb. efficiency, the lightest dose ( $10 \text{ gm/m}^3$  or 220 gm total fuel) will start to spill combustion products out of the room in 60 seconds (or at 132 gm). Higher doses will lose even more products, so we will not calculate them further without making other provision.

Two solutions to the problem of retaining the products in the room:

- (1) Transfer heat out of the room. It will make a major difference whether this is done in the early plume or at the boundaries of the room.
- (2) Make the fire some distance above the floor (shorter plume--less entrainment).

Let's try solution (2) and see what height the fire should be so that the ceiling layer hits the floor at the end of burning.

3.0 Calculate the effect of putting the fire at various heights above the floor--this will increase the room filling time. Assume the burning rate remains at  $2.2 \text{ gm/sec}$ . Then the longer filling time allows more fuel to be used.

Method: Assume various values of the fire height,  $\Delta H$ , calculate total filling time from  $\tau + \Delta\tau$  (Zukoski, section 6). Interpolate for 20, 30, 40  $\text{gm/m}^3$  (440, 660, 880 gm fuel) to find necessary  $\Delta H$  so as not to drive combustion products out of the floor vent.



Table 3.1

$\Delta H/H$	H	$\Delta H$	Q	$Q^*(H)$	T(y=0)	$\Delta T(\Delta H)$	$T_{Total}$ sec	Fuel Burned gms
0.0	2.44	0	49 Kw	.0058	60	0	60	124
0.1	2.20	0.24	49 Kw	.006	70.6	15.3	85.9	189
0.2	1.95	0.49	49 Kw	.005	68.7	27.4	96.2	211
0.3	1.57	0.87	49 Kw	.014	64.1	32.9	92.1	213
0.4	1.46	0.98	49 Kw	.017	61.7	41.3	103	226
0.5	1.22	1.22	49 Kw	.027	53.5	41.1	94.5	207
0.6	0.97	1.57	49 Kw	.048	46.4	39.1	85	187

$$H + \Delta H = 2.44 \text{ m,}$$

$$\Delta H = 2.44 - H$$

$$Q = 49 \text{ Kw} \approx 2.2 \text{ gm/sec} \approx 11,700 \text{ cal/sec}$$

$$Q^* = Q/\rho_c C_p T_c \sqrt{gH} H^2$$

$$= \frac{11,700 \text{ cal}}{\text{sec}} \frac{\text{m}^3}{1300 \text{ gm}} \frac{\text{gm}^\circ\text{K}}{0.24 \text{ cal}} \frac{\sqrt{9.8}}{273^\circ\text{K}} H^{1/2} H^2$$

$$(3.13)$$

$$= 4.4 \times 10^{-2} / H^{1/2}, = \frac{0.044}{\sqrt{H} H^2}$$

Table 3.2: Calculations for Table 3.1

$\sqrt{H}/H$	H meters	$\sqrt{H}$	$\sqrt{H} H^2$	$Q^*$	$Q^{1/3}$	$Q^{1/3} \tau$	$\tau/T$	$T_{sec}$	$\Delta t_f$	$\Delta T_f$ sec	$T_{Total}$ sec
						at y=0					
0.1	2.2	1.48	7.16	0.006	.182	14	1.09	70.6	16.6	15.3	85.9
0.2	1.95	1.40	5.32	0.008	.200	12.5	0.91	68.7	25	27.4	96.2
0.3	1.57	1.26	3.11	0.014	.240	10	0.65	64.1	21.4	32.9	97.1
0.4	1.46	1.21	2.58	0.017	.256	9	0.57	61.7	23.5	41.3	103
0.5	1.22	1.105	1.64	0.027	.300	7.2	0.45	53.3	18.5	41.1	94.5
0.6	0.97	0.982	0.92	0.048	.364	5.4	0.32	46.4	12.5	39.1	85

$$\tau/T = \frac{\sqrt{g}}{\sqrt{H}} \frac{H^2}{S} = \frac{\sqrt{9.8}}{\sqrt{11}} \cdot \frac{H^2}{(3.05)^2} = \frac{3.13}{9.3} \frac{H^2}{\sqrt{H}} = 0.335 \frac{H^2}{\sqrt{H}}$$

$$T_{y=0} = (Q^{1/3} \tau) \frac{1}{Q^{1/3}} \frac{1}{\tau/T} = \frac{(Q^{1/3} \tau)_{v=0}}{Q^{1/3} (\tau/T)}$$

$$\Delta \tau_f = \frac{\Delta H}{H} \cdot \frac{1}{Q^*(H)}$$



#### 4.0 Summary

From table 3.1, to burn 220 gm without losing products out the floor leak, place the fire about 0.98 meters off the floor.

Table 3.1 shows that we cannot burn much more and not lose products unless, of course, the combustion efficiency is lower than 80%, or heat is lost to the walls, or both.

# Development of a Stratified Ceiling Layer in the Early Stages of a Closed-room Fire

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A simple analytical model has been developed to determine the time required for a room to fill with products of combustion from a small fire. The room is assumed to be closed except for small openings at either the floor or ceiling level and the assumption is made that the leak is large enough to allow the transient pressure term in the energy equation to be neglected. Products of combustion are assumed to occupy a layer next to the ceiling and the model predicts the growth of the thickness and the mean density of this layer as a function of time. The analysis shows that times required to fill a typical room are small. For example, a typical bedroom fills with products from a 20 kW fire in several minutes. The time required to fill a room and the mean density of ceiling layer are determined in terms of fire size, room geometry, leak position, fire elevation and geometry.

## 1. INTRODUCTION

When heat is added to an ideal gas in a fixed volume, the pressure must increase in response to the temperature rise since the average density must remain fixed. In a building fire situation, the rate of pressure rise is often kept very small by gas leaks through openings in the walls of the building such as cracks around windows and doors.

Under circumstances for which leaks do keep the rate of pressure rise to a negligible value, we are interested in the time required for the gas remaining in the volume to be contaminated and heated by mixing with the products of combustion from a fire. In the following paragraphs, we will examine this problem for a very simple example. The fire will be treated as a point source of heat with a specified strength. We will restrict our examination to a volume composed of a single room with a horizontal ceiling layer of hot gas formed under the ceiling. This layer may contain a nonuniform temperature distribution but we will be concerned only with its average temperature or density. During the progress of the fire the thickness of this layer will grow in time and we will be interested in predicting when the lower boundary will reach the floor level. We are also interested in the average density in this layer as a function of time.

This two layer model is appropriate for a fire of small geometrical area which is burning in a room of much larger floor area prior to flashover of the room.

Our purpose is to illustrate the general order of magnitude of the time involved and the manner in which various parameters influence this time by looking at cases which are mathematically very simple. The complete problem is a special case of a room fire and numerical integration techniques are available if more accuracy and detail is required.

## 2. PRESSURE RISE IN A CLOSED VOLUME

In order to justify the constant pressure assumption used in examining a leaky room-fire consider the pressure increase produced by a fire in a closed volume. For many interesting situations the pressure will rise to a quasi-steady state value in a time which is short compared with the time scale of other interesting events. To illustrate this conclusion consider the internal energy balance for a perfectly closed volume  $V$  containing an ideal gas which is heated at a rate  $\dot{Q}$ :

$$\frac{d}{dt} \left( \int_V (\rho c) dV \right) = \dot{Q}/T \quad (1)$$

The mass balance is

$$\frac{d}{dt} \left( \int_V (\rho) dV \right) = 0 \quad (2)$$

If we assume that specific heats are constant, the gas is ideal, that hydrostatic pressure differences are negligible and that heat addition rate,  $\dot{Q}$ , is constant, then Eqs (1 and 2) can be combined and integrated to give

$$\frac{P - P_a}{P_a} = \frac{\dot{Q}t}{\rho_a V C_V T_a} \quad (3)$$

Here, the subscript  $a$  designates the ambient conditions before the heat addition starts. A numerical example is of interest. Consider a room with a volume of  $28.12 \text{ m}^3$  which contains a fire of  $100 \text{ kW}$  heat input rate. Then  $(\dot{Q}/\rho_a V C_V T_a) = 0.007 \text{ s}^{-1}$  and consequently the pressure rises by about  $0.07 \text{ atm}$  in  $10 \text{ s}$ . A pressure differential of this amount across a window of  $0.6 \text{ m}^2$  will produce a total load of about  $3500 \text{ Newtons}$  which (a) might be enough to destroy the window and (b) will produce a velocity of about  $1.20 \text{ m s}^{-1}$  in room temperature air through a leak. Both effects are large and presumably would lead to sufficient leaks to keep further pressure rise from occurring.

This example suggests that quasi-steady pressure within a burning room is a reasonable assumption. A quantitative measure of leak areas needed for a given room and fire is given in Section 10.

### 3. HEAT ADDITION TO A FIXED VOLUME WITH LEAKS

Consider a fixed volume in space containing an ideal gas to which heat is added. Note, that here the fire is considered to be a source of heat alone and the mass of fuel is neglected. Mass is allowed to leave the volume such that the work done by the rate of change of pressure within the volume absorbs a negligible fraction of the heat input.

We will show here that under these circumstances the enthalpy flux produced by this mass flux is equal to the heat addition rate and that the enthalpy of the gas remaining in the volume is constant regardless of the distribution of temperature within the volume.

The energy equation for the mass within the volume  $V$  can be written as

$$\frac{d}{dt} \left( \int_V \rho v e dV \right) + \int_S (\rho v \cdot dSh) = \dot{Q} + \dot{Q}_c \quad (4)$$

where  $e$  is the internal energy,  $V$  is a volume fixed in space,  $S$  is the surface of the volume,  $\rho v \cdot dS$  is local mass flux through an element of surface area due to vector velocity  $v$ ,  $h$  is enthalpy of gas crossing  $S$ ,  $\dot{Q}$  is rate of heat addition by the 'fire,' and the last term  $\dot{Q}_c$  is the rate of heat conduction across the boundary into the room.

If we now combine Eqn (4) with Eqn (2) for continuity and the definitions of internal energy and enthalpy for an ideal gas:

$$e = C_v(T - T_r) \quad (5)$$

and

$$h = C_p(T - T_r) \quad (6)$$

We can rewrite Eqn (4) as

$$\frac{C_v V}{R} \left( \frac{dP}{dt} \right) + \int_S (C_p T) \rho v \cdot dS = \dot{Q} + \dot{Q}_c \quad (7)$$

Here  $T_r$  is a reference temperature, and  $C_p$  and  $C_v$  are suitably chosen values of specific heats of the gas at constant pressure and constant volume. We have also made use of the approximation that the specific heats of the gas are constant and that the gas follows the state equation for the ideal gas.

$$P = \rho RT \quad (8)$$

When the transient pressure term in (7) is small compared with the heat term, we can neglect the effect of pressure transient. Further discussion of the conditions under which this pressure term must be included in the equation is given in Section 10 of this paper.

When the first term of (7) can be neglected, we get

$$\int_S (\rho v \cdot dS) (C_p T) = \dot{Q} + \dot{Q}_c \quad (9)$$

If we restrict the outflow to a single point where conditions are uniform in space, then

$$\int_S (\rho v \cdot dS) C_p T = \dot{m}_e C_p T_e$$

where  $\dot{m}_e$  is mass flow at the exit (e) and  $C_p T_e$  is the local gas enthalpy. Thus (9) may be written as

$$\dot{m}_e C_p T_e = \dot{Q} + \dot{Q}_c$$

or, if there are a number of leaks

$$\sum_i \dot{m}_i C_{pi} T_i = \dot{Q} + \dot{Q}_c \quad (10)$$

will hold. When conduction is ignored,  $\dot{Q}_c$  will be zero and we get the particularly simple result that the enthalpy flux from the volume equals the heat addition regardless of temperature distribution within the volume:

$$\sum_i \dot{m}_i C_{pi} T_i = \dot{Q} \quad (11)$$

### 4. ROOM PROBLEM

We are interested in determining the time required to fill a room with products of combustion from a fire. We want to make a simple calculation and to estimate the effects of leaks on this process.

The fire is treated as a point source of heat addition, the fuel flow rate is neglected and the plume above the fire is treated in the usual Boussinesq manner. The ceiling layer is taken as an adiabatic region. Because we are interested in predicting the level of the ceiling layer (Y in Fig. 1), we need *not* make assumptions concerning the degree of mixing in this region. Symbols are defined in Fig. 1. The lower boundary of the ceiling layer is assumed to be horizontal. We want to predict the downward motion of this boundary.

In the following analysis, the volume of the plume and fuel mass flow rate are ignored.

#### Floor leak case

In this first example let the leak be at the floor level so that only uncontaminated gas escapes. A mass balance for the cold region is given by

$$\frac{d}{dt} (\rho_c Y S) + \dot{m}_p = 0 \quad (12)$$

Here  $S$  is the area of floor of the room;  $\rho_c Y S$  is the mass of uncontaminated gas;  $\dot{m}_p$  is the rate at which mass flows out of the cold region due to entrainment into the

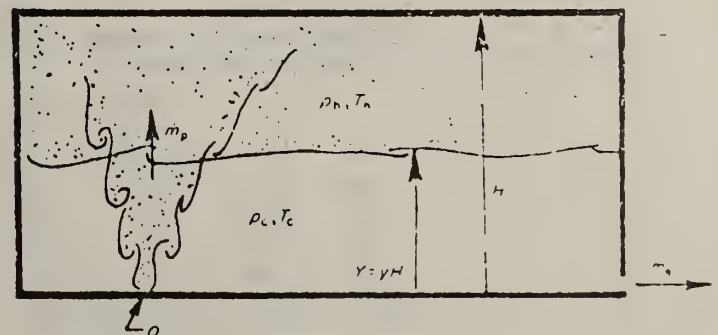


Figure 1. Room fire model, floor level leak.



plume; and  $\dot{m}_p$  is the mass lost through leaks from the cold region. If we choose  $r \equiv Y/H$ , (13) becomes

$$(\rho_c H S) \left( \frac{dr}{dt} \right) + \dot{m}_c + \dot{m}_p = 0 \quad (14)$$

The mass flow rate at the leak, which is the only leak, is given by (12) as

$$\dot{m}_c = \dot{Q} / C_p T_c = \dot{Q} / C_p T_e$$

or

$$\dot{m}_c = (\dot{Q} / \rho_c C_p T_c \sqrt{g H / H^2}) (\rho_c \sqrt{g H / H^2})$$

If we use a nondimensional fire heat input parameter,

$$Q^* \equiv \dot{Q} / \rho_c C_p T_c \sqrt{g H / H^2}$$

then

$$\dot{m}_c = Q^* \rho_c \sqrt{g H / H^2} \quad (15)$$

Previous analysis has given a reasonable estimate of plume mass flux as

$$\dot{m}_p = (Q^*)^{1/3} (\rho_c \sqrt{g H / H^2}) \alpha_1^{5/3} \quad (16)$$

where  $\alpha_1$  is a collection of constants  $(\pi C_p C^2)$  whose product is about  $(1/5.4)^{1/3}$ . This result is also discussed briefly in the Appendix. Collecting these items, (15) and (16), we can rewrite (14) as

$$\frac{dr}{d\tau} + Q^* + \alpha (Q^*)^{1/3} \theta_0^{5/3} = 0 \quad (17)$$

where  $\tau$  is a nondimensional time defined as

$$\tau \equiv t (\sqrt{g/H}) (H^2/S) \quad (18)$$

Here  $S$  is area of the floor of the room and  $H$  is the height of the room. The second term in (17) is the contribution of the leak and the third, is due to plume entrainment.

The integration of (18) is easily accomplished by numerical techniques when  $Q^*$  is a constant. Thus

$$\tau = \int_0^1 \left[ \frac{dr}{Q^* + \alpha (Q^*)^{1/3} \theta_0^{5/3}} \right] \quad (19)$$

Values of  $\tau$  versus  $r$  are given in Table 1 and also in Fig. 2 for the three 'fires'. The parameter  $(\tau(Q^*)^{1/3})$  is used in presenting these results because of its convenience in the next example. Note that times here are short. For example, if our room is 2.44 m (8 ft) wide by 2.44 m high by 9.75 m long and if  $\dot{Q} = 95$  kW, then  $Q^* = 0.01$ ;  $t/\tau = 2$  s;  $\tau|_{r=0} = 51$  and the corresponding time  $t$  is 102 s. Thus the ceiling layer would reach the floor level in less than 2 min. Dependence on  $Q^*$  is strong when  $Q^*$  is larger than 0.01.

Table 1. Dependence of times required to fill upper half of a room on fire size and room area for  $H_r = 2.44$  m

S	$\sqrt{H_r S / g H^2}$	Q	Q*	Floor Ceiling	
				$t _{r=1/2}$	$t _{r=1/2}$
8.9 m <sup>2</sup>	0.75 s	100 kW	0.010	11 s	16.5 s
36	3	100	0.010	45	66
143	12	100	0.010	178	264
36	3	20	0.002	97	117
36	3	100	0.01	45	66
36	3	500	0.05	16.5	40

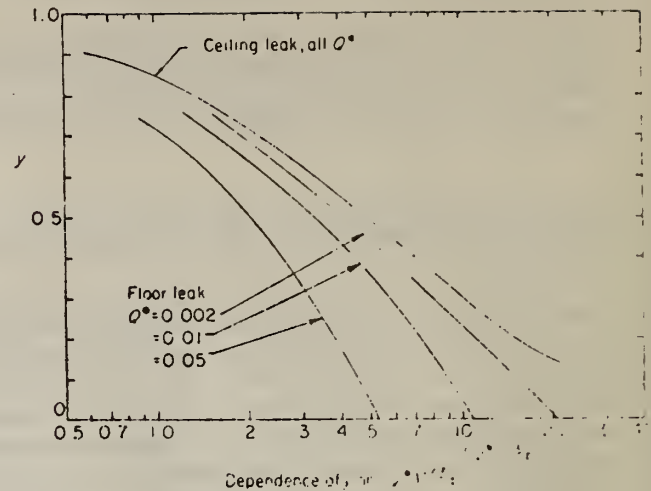


Figure 2. Dependence of ceiling layer height on time and heat input rate.

### Ceiling exit case

If the exit is in the ceiling layer, the problem is even simpler. For this case, no mass leaks from the cold region, and the equation for mass balance of the cold fluid, Eqn (17), reduces to

$$\frac{dr}{d\tau} + \alpha (Q^*)^{1/3} \theta_0^{5/3} = 0 \quad (20)$$

which can be immediately integrated to give

$$r = \left[ 1 + \left( \frac{2}{3} \right) \alpha (Q^*)^{1/3} \tau \right]^{-3/2} \quad (22)$$

Values of  $\tau$  required for the ceiling layer to reach a specified  $\theta$  are given in Table 2 and Fig. 2. Note that  $[(Q^*)^{1/3} \tau]$  is clearly the appropriate scaling parameter for this case.

Consider the room discussed above with  $Q^* = 0.01$  and  $t/\tau = 2$  s. At  $r = 1/2$ ,  $\tau = 22$  and  $t = 44$  s, and when  $r = 0.1$ ,  $\tau = 136$  and  $t = 272$  s. Note that  $r$  approaches zero asymptotically and that for all  $r$ , values of  $\tau$  are larger for the ceiling leak than for the floor leak. However, as  $Q^*$  approaches zero, Eqns (17 and 21) begin to converge as long as  $r > 0$ . For a very weak fire, say  $Q^* = 0.0005$ , e.g. ( $\dot{Q} = 5$  kW,  $H = 2.44$  m), the differences are reduced to a few percent at  $r = 1/4$  and can be ignored.

The result given in Eqn (22) is identical to that obtained by Baines and Turner<sup>2</sup> for a similar problem involving an incompressible flow in which the effects of leaks could be ignored. The results are similar because in both calculations the density of the cold layer below the interface is assumed to be constant and consequently only the plume entrainment enters the problem. Experiments of Baines and Turner<sup>2</sup> verified the accuracy of the equation for values of heat addition (buoyancy flux in their case) corresponding to  $2 \times 10^7$  to  $3 \times 10^8$ .

For the examples discussed here,  $\tau|_{r=1}$  depends on  $Q^*$  which is fixed by the heat input rate and  $H$ . Thus  $\tau$  is a function of fire heat input rate and room height but is independent of room area. Equation (18) shows that the time  $t$  will scale linearly with floor area  $S$ . Several examples are shown for both ceiling and floor leak cases in Table 1.

Scaling with room height is more complex. If we compare rooms with fixed floor area and heat input,



Table 2. Dependence of ceiling layer height and mean density on dimensionless time for floor and ceiling leaks

Floor leak							Ceiling leak All $Q^*$ ( $Q^*$ ) <sup>1/2</sup>
$\gamma$	$Q^* = 0.002$		$Q^* = 0.01$		$Q^* = 0.05$		
	$\tau$	$\bar{\rho}_h/\rho_c$	$\tau$	$\bar{\rho}_h/\rho_c$	$\tau$	$\bar{\rho}_h/\rho_c$	
1.0	0	--	0	--	0	--	0
0.75	12.2	0.99	6.0	0.76	2.4	0.53	1.71
0.50	32.2	0.87	14.9	0.70	5.5	0.45	4.76
0.25	71.5	0.81	29.1	0.61	9.4	0.37	12.31
0.00	164	0.67	51	0.49	14.2	0.29	$\infty$

and change room height, then both  $Q^*$  and the  $\sqrt{g/H}$  ( $S/H^2$ ) parameter will change. The effect of changing  $H$  on  $\tau$  for a fixed  $\dot{Q}$  is shown in Table 3 for the floor leak case and for  $\gamma = 1/2$  and 0. Note that increasing  $H$ , decreases  $Q^*$ , increases  $\tau$ , has a mixed effect on  $\tau^{1/2}$ , and increases  $\tau^{1/4}$ . Thus, a room with a high ceiling fills up only slightly slower than a room with a lower ceiling but the same floor area.

### 5. CEILING LAYER TEMPERATURE ESTIMATES; FLOOR LEAK

Consider the case for which the leak is at the floor level. An energy balance for an adiabatic ceiling layer gives

$$\int_0^t \dot{Q} dt = \int_0^1 \rho (SH) d\gamma (h - h_c) \quad (23)$$

where  $h$  is the enthalpy of the gas. Thus, if  $\dot{Q}$  is constant we find

$$\dot{Q} t = \int_0^1 \rho (SH) (d\gamma) (C_p T - C_p T_c)$$

If we use again  $pT = p_c T_c$  from the ideal gas equation of state, we find that

$$\dot{Q} t = p_c C_p T_c (SH) (1 - \gamma) - C_p T_c (SH) \int_0^1 \rho d\gamma \quad (24)$$

when we define the mean ceiling layer density as

$$\bar{\rho}_h SH (1 - \gamma) \equiv \rho (SH) d\gamma$$

we can rearrange Eqn (24) to give

$$(\rho_h/\rho_c) = (1 - (Q^* \tau)/(1 - \gamma)) \quad (25)$$

Values of  $\rho_h/\rho_c$  are listed in Table 2 for the three examples discussed there. Note, that even for the smallest fire, density differences are appreciable; certainly for the larger two fires the Boussinesq approximation is not satisfactory.

Even for the weak fire described above, ( $\dot{Q} = 5$  kW,  $H = 2.44$  m, and  $Q^* = 0.0005$ ) the average density in the ceiling layer when  $\gamma = 0$  is about 10% below that of the cool gas.

Table 3. Dependence of time required to fill a room on room height for  $\dot{Q} = 100$  kW

$H$	$Q^*$	$S$	$\tau; \gamma = 1/2$	$\tau; 1/2$	$\tau; \gamma = 0$	$\tau; 0$
1.28 m	0.050	143 m <sup>2</sup>	5.5	174s	14.2	448s
2.44	0.010	143	14.9	179	51	609
4.63	0.002	143	32.2	148	164	750

In the cases for which an impurity level, say carbon monoxide, can be related to the heat input rate by a relationship such as

$$\dot{m}_{co} = C (\dot{Q} C_p T_c)$$

the mass fraction of impurity in the ceiling layer will be

$$(\text{mass fraction}) = C Q^* \tau (1 - \gamma) \quad (26)$$

Equation (25) holds for the floor leak only. Similar results can be developed for the ceiling leak case but require numerical integration and, more important, some assumption about the temperature profile in the ceiling layer.

### 6. FIRE LEVEL ABOVE FLOOR

The level of the fire above the floor has interesting effects. For the ceiling leak case, the interface level will asymptotically approach the fire level and gas beneath the fire will remain uncontaminated, since in our model cold gas is only removed from the room by entrainment into the plume. However, for the floor level leak, the interface will still reach the floor, although Eqn (17) is not valid after the interface reaches the fire level. After that time, plume entrainment no longer enters the problem. Thus, Eqn (17) is reduced to

$$\frac{d\gamma}{d\tau} + Q^* = 0$$

We use the same notation to describe the geometry for the room above the fire as above, and add a distance  $\Delta H$  to the room below the fire (see Fig. 3). We assume that the ceiling layer interface reaches the level of the fire  $\gamma = 0$  at a time  $\tau_1$ , and that an additional period

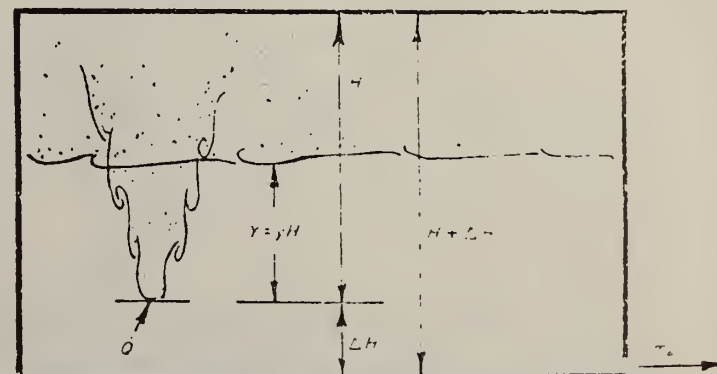
Figure 3. Notation for room with fire elevated at  $\Delta H$  from floor

Table 4. Effect of fire elevation on time required to fill a room for  $H + \Delta H = 2.44$  m

$\Delta H$ H	H	S	Q	$Q^*/H$	$\tau(y=0)$	$\Delta\tau_f$	$t_f$	t
0	2.44 m	36 m <sup>2</sup>	100 kW	0.01	51	0	3 s	153 s
0.25	1.83	36	100	0.02	29	16.7	4.62	211

$\Delta\tau_f$  is required to fill the height below the fire level. Integration of the above equation leads to

$$\Delta\tau_f = \frac{\Delta H}{H} \frac{1}{Q^*/H}$$

where  $Q^*$  is based on  $H$ , the distance from the fire to the ceiling.

To illustrate the solution compare the two examples described in Table 4. In the second, the fire is elevated 0.61 m (2 ft) above the floor and in this case the effective value of ceiling height  $H$  is 1.63 m (6 ft) rather than 2.44 m (8 ft). Hence, for the same heat input rates,  $Q^*$  is larger for the second example and  $\tau(y=0)$  is smaller. However, to determine the value of  $\tau$  at which the interface reaches the floor level we must add the  $\Delta\tau_f$  term so that for the second example the value of  $\tau$  required for the ceiling layer interface to reach the floor is  $29 + 17 = 46$ . This is converted to the dimensional time as usual but again  $H$  is used, not the room height which is  $(H + \Delta H)$ . The time for the elevated fire is about 40% greater than for the floor level case.

Finally, a leak at an intermediate level can be studied as a combination of our two extreme cases. The motion of the interface will be described by Eqn (17) until the interface reaches the level of the leak and by Eqn (21) thereafter.

## 7. NON-CONSTANT HEAT INPUT RATES; CEILING LEAK

If the heat input is not constant, Eqn (21) for the ceiling leak can still be integrated directly to give

$$r^{2/3} - 1 = \frac{2}{3} \int_0^t \alpha Q^* (\tau)^{1/3} d\tau$$

In order to examine the impact of nonuniform heating rate let  $Q^* = q\tau$ , that is, consider a linear increase in

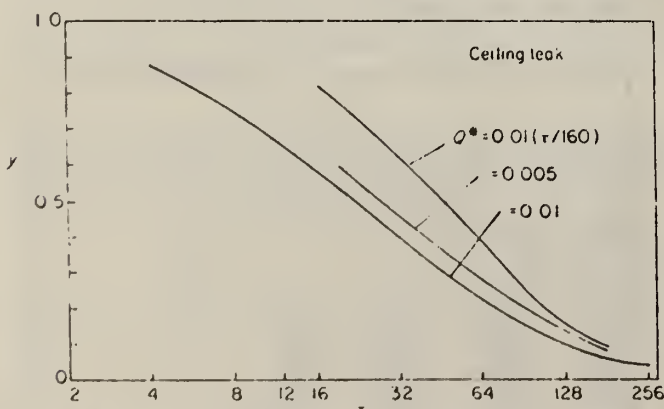


Figure 4. Dependence of ceiling layer height on time for time dependent heat input.

fire heat input. Then the above equation can be evaluated as

$$r = \left[ 1 + \frac{\alpha}{2} (q)^{1/3} \tau^{1/3} \right]^{3/2} - \left[ 1 + \frac{\alpha}{2} Q^* (\tau)^{1/3} \right]^{3/2} \quad (27)$$

Consider three numerical examples. First and second, fires with constant heat inputs given by  $Q^* = 0.01$  at  $\tau = 160$ . Figure 4 gives values of  $r$  for these examples up to  $\tau = 160$ . Note that  $\tau = 160$ , total heat added in cases two and three is equal, and that the values of  $r$  reached at this time are almost equal. Thus, the total heat addition rather than details of the rates involved is the critical factor.

## 8. HEAT LOSSES TO THE WALLS; FLOOR LEAK

Transfer of heat to the walls from the ceiling layer region will modify the leak rate required to keep the total room enthalpy constant. Equation (17) for the floor leak example can be rewritten as

$$\frac{dr}{d\tau} + (1 - \lambda) Q^* + \lambda (Q^*)^{1/3} r^{5/3} = 0 \quad (28)$$

where  $\lambda$  is the fraction of heat addition by the fire which is lost by conduction to the walls. Note that if  $\lambda = 1$ , the second term in the above equation drops out and the equation becomes identical to that used for the ceiling level leak. Hence, the effect of heat loss will be to make the  $r$  versus  $\tau$  curve for a floor leak case with heat loss lie between the adiabatic floor level leak case and the ceiling level leak case shown in Fig. 2. Clearly the effects of heat loss will be larger for the larger values of  $Q^*$ .

To illustrate these effects, values of  $r$  are shown below in Table 5 as a function of  $r$  with  $Q^*$  and  $\lambda$  as parameters. Comparison of the first two columns and last two columns for  $\tau$  shows that when  $\lambda = 1/2$  values of  $\tau$  are increased by about a factor of 1.5 at  $r = 0$  and by smaller factor for  $r > 0$ . Thus, if 50% of the heat input is lost to the walls, a 50% or less increase will occur in  $\tau$ . The  $Q^* = 0.01$ ,  $\lambda = 0.5$  case and  $Q^* = 0.005$ ,  $\lambda = 0$  case have the same net heat inputs. However, the former case has a stronger plume entrainment and consequently smaller  $\tau$  values. Similar results are found by comparing  $Q^* = 0.01$ ,  $\lambda = 0.8$  with  $Q^* = 0.002$ ,  $\lambda = 0$ .

For the ceiling leak example, conduction losses do not enter the problem at all since the only mass loss mechanism for the uncontaminated gas is plume entrainment.

## 9. LINE FIRE EXAMPLE

A similar development can be carried out for a two-dimensional or line plume.<sup>1</sup> For this configuration, the entrainment in the plume at an elevation  $(y) = (H)$

Table 5. Effect of heat loss, fire size and ceiling layer level on dimensionless time

$Q^* =$	0.01			0.005	0.002	0.05	
$\lambda =$	0	0.5	0.8	0	0	0	0.5
$\gamma = 1$	0	0	0	0	0	0	0
0.75	6	6.8	7.5	7.5	12.2	2.4	3.2
0.50	15	18	20	24	32.2	5.5	7.6
0.25	29	38	47	54	72	9.4	14
0.0	51	78	127	126	164	14	23

Table 6. Dependence of dimensionless time on ceiling level height, fire size and leak position for a line fire

$Q_2^* =$ $\gamma$	Ceiling leak		Floor level leak							
	All $Q_2^*$ $(Q_2^*)^{1/3} \tau$	$0.01$ $\tau$	$0.002$		$0.01$		$0.05$			
			$(Q_2^*)^{1/3} \tau$	$\tau$	$(Q_2^*)^{1/3} \tau$	$\tau$	$(F/\rho_c)$	$(Q_2^*)^{1/3} \tau$	$\tau$	
1.00	0	0	0	0	0	0	0.92	0	0	
0.75	0.57	2.7	0.555	4.4	0.514	2.4	0.91	0.436	1.19	
0.50	1.37	6.4	1.33	10.6	1.21	5.6	0.90	1.00	2.71	
0.25	2.74	12.7	2.60	21	2.29	11	0.88	1.78	4.83	
0.10	4.55	21	4.12	33	3.46	16	0.85	2.46	6.1	
0.05	5.99	28	5.07	40	4.07	19	0.83	2.75	7.5	
0.0	7.4	31	6.97	55	4.93	23	0.81	3.09	8.4	
$\gamma =$	0					1				

above a plume of length  $\mathcal{L}$  and total heat input  $\dot{Q}$  is given by

$$\dot{m}_p = (\sqrt{\pi} C_{v2} C_{12}) \rho_c \sqrt{gH} (H/\mathcal{L}) (Q_2^* \{H\})^{1/3} (r)$$

where

$$Q_2^* \{H\} = \dot{Q} / [\rho_c C_p T_c \sqrt{gH} H/\mathcal{L}]$$

$$(\sqrt{\pi} C_{v2} C_{12}) \equiv \alpha_2 = (0.51)$$

Note that here  $\dot{m}_p/r$ .

The mass balance for the cold air layer can be written as

$$\frac{dr}{d\tau} + \gamma Q_2^* + \alpha_2 (Q_2^*)^{1/3} r = 0$$

where

$$\tau = t\sqrt{g}/H \quad (\mathcal{L}/H/S)$$

Here,  $\gamma=1$  for floor leak and  $\gamma=0$  for ceiling leak. Integration of this equation leads to

$$(Q_2^* \{H\})^{1/3} \tau = \frac{1}{\alpha} \ln \left\{ \frac{\alpha + \gamma (Q_2^*)^{2/3}}{\alpha r + \gamma (Q_2^*)^{2/3}} \right\}$$

or approximately

$$(Q_2^* \{H\})^{1/3} \tau = 2 \ln \left\{ \frac{1 + 2\gamma (Q_2^*)^{2/3}}{1 + 2\gamma (Q_2^*)^{2/3} r} \right\}$$

A number of examples are given in Table 6. Note that if the room involved is 2.44 m high by 2.44 m wide by 9.75 long and if  $\mathcal{L}=2.44$  m,  $(t/\tau)=2$  s.

The density ratio can be calculated easily for the floor level leak case and the result is the same as Eqn (26) which was obtained for the axisymmetric plume. The ratio for  $\tau=0$  and  $r=1.0$  is the value obtained at the start of the gas flow,  $t=0+$ , when the plume first reaches the ceiling. The line and axisymmetric fires are compared in Fig. 5 for floor leak cases with  $Q^*=0.01$ .

The time required for the interface to reach a given level is much shorter for the line fire. Thus, fire geometry

can be a very important parameter. The difference between the ceiling and floor leak positions are similar for the line plume and axisymmetric plumes.

## 10. NONCONSTANT PRESSURE

We want to return now to examine in more detail the relationship between room size, heat input, and our assumption that the rate of change of pressure is negligible.

Equation (8) can be written as

$$\left( \frac{C_p V}{R} \right) \frac{dP}{dt} + m_c C_p T_c = \dot{Q} \quad (27)$$

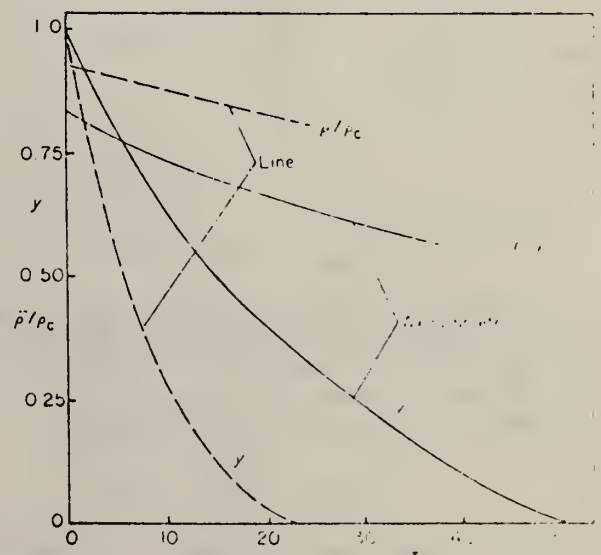


Figure 5. Dependence of mean density ratio and ceiling layer height on dimensionless time for axisymmetric and line fires for  $Q^*=0.01$ .



for a room with a single exit, a time dependent pressure, and no conduction; further let  $\dot{Q}$  be constant. For simplification, let the leak be at the floor level and ignore any adiabatic heating of the uncontaminated gas by compression. Then density and temperature at the leak are constant and have the ambient values,  $\rho_e$ ,  $T_e$ . If the heat addition starts at  $t=0$ , the pressure within the room will rise until a steady state is reached. Steady state condition implies the pressure is constant so that (29) becomes

$$\dot{m}_{es} C_p T_e = \dot{Q} \quad (30)$$

where the subscript *s* designates the steady state. If the flow through the hole resembles that through an orifice, the velocity  $v_e$  is related to the pressure difference across the leak ( $P - P_a$ ) by

$$\frac{1}{2} \rho_e v_e^2 = (P - P_a) \equiv \Delta P$$

or

$$v_e = \sqrt{2\Delta P / \rho_e}$$

and

$$v_{es} = \sqrt{2\Delta P_s / \rho_e} \quad (31)$$

Here  $P_a$  is the ambient pressure and  $\rho_e$  is taken to be the cold air density  $\rho_e$ . The corresponding mass fluxes are

$$\begin{aligned} \dot{m}_e &= \sqrt{2\rho_e \Delta P} A_e = \rho_e v_e A_e \\ \dot{m}_{es} &= \sqrt{2\rho_e \Delta P_s} A_e = \rho_e v_{es} A_e \end{aligned} \quad (32)$$

where  $A_e$  is the effective leak area. For the steady case we can use (30), (31) and (32) to express  $\Delta P_s$  as

$$\Delta P_s = \left( \frac{\dot{m}_{es}}{A_e} \right)^2 \frac{1}{2\rho_e} = \left( \frac{\dot{Q}}{C_p T_e A_e} \right)^2 \frac{1}{2\rho_e}$$

or

$$\Delta P_s = \frac{1}{2} \rho_e (gH_e) (H_e^2 / A_e)^2 (Q^*)^2 \quad (33)$$

We can use this result, Eqn (30) and the definitions,

$$\begin{aligned} X &= \Delta P / \Delta P_s \\ t_e &= (\gamma/2) (\sqrt{gHH/a_0^2}) Q^* (H_e^2 / A_e) (S/H^2) \\ \theta &= t/t_e \end{aligned} \quad (34)$$

and  $a_0^2 = \gamma R T_e$

to rewrite (29) as

$$\frac{dX}{d\theta} + X^{1/2} = 1 \quad (35)$$

The solution for (35) is

$$\theta = 2 [\ln(1/(1 - X^{1/2})) - X^{1/2}] \quad (36)$$

The dimensionless time required for the pressure to reach its equilibrium value  $X = \Delta P / \Delta P_s = 1$  is, of course, infinite. However, if we pick a value close to 1, say  $X = 0.86$ , we can obtain a good idea of the time required to approach the equilibrium value. From (36),

$$\theta_i(X = 0.86) = \frac{t_i(0.86)}{t_e} = 3.46$$

Thus, in a time (3.46  $t_e$ ) the pressure will rise to 86% of its equilibrium value.

We are interested in comparing this time with that required for the ceiling layer to reach the floor ( $t_i, y=0$ )

Table 7. Effect of leak and fire size on pressure transient, and gas velocity in the leak

$Q^*$	$A_e/H_e^2$	$\tau_i, y=0$	$\left\{ \begin{array}{l} \tau_i, X=0.86 \\ \tau_i, y=0 \end{array} \right\}$	$\Delta P_s/P_a$	cs
0.002	0.003	164	0.0006	$6.6 \times 10^{-4}$	11 ft/s
0.01	0.003	51	0.011	$1.6 \times 10^{-4}$	5.3
0.05	0.003	14.2	0.20	$4.1 \times 10^{-4}$	26.7
0.01	0.003	51	0.011	$1.6 \times 10^{-4}$	5.3
0.01	0.002	51	0.026	$3.7 \times 10^{-4}$	8.6
0.01	0.001	51	0.10	$1.5 \times 10^{-4}$	16.0

for the corresponding floor leak case. If this ratio is small, the quasi-steady state solution discussed in previous paragraphs will be useful. The ratio is

$$\frac{t_i(X=0.86)}{t_i(y=0)} = \frac{\theta_i(X=0.86)}{\tau_i(y=0)} = \left[ \frac{\gamma}{2} \left( \frac{gH}{C_p a_0^2} \right) \left( \frac{H_e^2}{A_e} \right) Q^* \right] \quad (37)$$

Note, that in none of the terms on the right does  $(S/H^2)$  appear either explicitly or implicitly. Hence, the floor area  $S$  does not enter the ratio of the times. Also note that  $Q^*$  increases,  $\tau_i, y=0$  decreases, and thus the ratio increases strongly with  $Q^*$ . Several examples are given below in Table 7 for floor leak case for which  $H_e = 2.44$  m, and  $a_0 = 360$  m/s.

For an 2.44 m high room a value of  $(1/C_p a_0^2) = 0.003$  gives a leak area of 168 cm<sup>2</sup> (28 in<sup>2</sup>). This is not an unreasonable value for a 10 m long hall with three doors. If each door is 1 m wide and has a 1.2 cm crack the leak area would be 150 cm<sup>2</sup>.

Note, in Table 7, that as  $Q^*$  increases the ratio of times increases rapidly. For  $Q^* = 0.002$  and 0.01, the ratio is small enough that the quasi-steady assumption is a good one. For  $Q^* = 0.05$  it is not as satisfactory. Hence, for a large fire, pressure changes will be more important. In all cases the pressure rise is so small that gas density and pressure are virtually unaffected.

The results presented above allow a quantitative determination to be made of the leak area required for a given room-fire configuration to insure that the constant pressure assumption be valid. The leak area required increases rapidly with the dimensionless heat addition parameter and scales as the room height squared.

## 11. SUMMARY

The room fill up times have been examined for rooms which have large enough leaks to make the quasi-steady pressure assumption reasonable, and for a number of other special conditions which simplified the analytic work and which were interesting limiting cases. Some interesting results are as follows:

1. The time depends on location of the leak and can be considerably (factor of two) shorter for a floor leak than a ceiling leak.
2. For a constant rate of heat input, fill up time varies linearly with room floor area and roughly as the 0.4 power of room height.
3. For the ceiling leak, the time is inversely proportional to the (1/3) power of the heat input parameter  $Q^*$  which is directly proportional to the fire heat input rate.



4. For geometrically similar rooms and fires with the same  $Q^*$  values, the time scales as the square root of the room height.
5. Heat loss to walls has an effect on the time for floor leak example, but none for the ceiling leak case.
6. A scheme is given to allow estimation of leak area required to make the quasi-steady pressure assumption a useful one.
7. The effects of positioning the fire above the floor can be treated for either leak position. Raising the fire will increase the fill up time for the floor leak and will also produce an uncontaminated layer below the fire level for the ceiling leak case.
8. The time required for the interface to reach the

floor is not strongly dependent on the time dependence of heat addition, but does depend on the total energy added.

9. Geometry of the fire has a strong influence on its density in the ceiling layer and on the time required to fill the room with products.

These results indicate that reasonable estimates of fill up time can be made without requiring precise definitions of heat input rates or heat losses to boundaries.

#### Acknowledgement

The work reported in this paper grew out of a discussion between the author and Dr. J. Quintiere and E. Rockett of the Fire Research Center, National Bureau of Standards and was carried out under a grant from the Fire Research Center.

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#### APPENDIX 1: NOMENCLATURE

##### List of symbols

$C_p$	Specific heat at constant pressure.
$C_v$	Specific heat at constant volume.
$e$	Internal energy.
$g$	Gravitational constant.
$h$	Enthalpy.
$H$	Room height.
$l$	Length of line fire.
$\dot{m}$	Mass flux.
$\dot{m}_e$	Mass flux at exit.
$\Delta P$	Pressure difference across leak.
$\dot{Q}$	Heat addition rate from fire.
$Q^*$	$Q/\rho_a C_p T_a \sqrt{g H H^2}$
$Q_e$	Heat conducted out of ceiling layer gas.
$q$	$Q^* \tau$ , see Eqn (27).
$R$	Gas constant.
$S$	Area of floor of room.
$S_e$	Area of opening.
$T$	Temperature.
$t$	Time.
$t_c$	Critical time.
$V$	Volume of room.
$v$	Velocity.
$v, dS$	Volume flux at opening.
$X$	$\Delta P/\Delta P_c$ , see Eqn (34).

$Y$	Height of lower edge of ceiling layer.
$y$	$Y/H$ .
$\alpha$	Dimensionless constant, see Appendix.
$\gamma$	$\gamma=1$ for floor leak, and $\gamma=0$ for ceiling leak; in Eqn (34) $\gamma$ is specific heat ratio equal to 1.40 for air.
$\theta$	$t/t_c$ , see Eqn (34).
$\lambda$	Heat loss factor, Fire heat input lost by conduction to walls.
$\rho$	Density.
$\bar{\rho}_u$	Mass averaged density of ceiling layer, see Eqn (25).
$\tau$	Dimensionless time $= t(\sqrt{g/H})/(H^2/S)$ .
$\Delta \tau_f$	Dimensionless time required to fill volume beneath fire level.

##### Subscripts

$a$	Ambient condition outside room.
$c$	Cool gas property in room.
$e$	Exit.
$i$	$i$ th exit.
$m$	Maximum value.
$p$	Plume property.
$r$	Reference value.
$s$	Steady state value.
$2$	Line fire parameter.

#### APPENDIX 2: PLUME PROPERTIES AND MASS BALANCE

The turbulent fire plume can be characterized by the following equations when density differences are small and when the elevation above the fire,  $Z$ , is large compared with the fire diameter:

$$\frac{\Delta T_m}{T_1} = \frac{\Delta \rho_m}{\rho_1} = C_T (Q^*)^{2/3} \quad C_T \approx 9.1 \quad (A1)$$

$$\frac{w_m}{\sqrt{gZ}} = C_v (Q^*)^{1/3} \quad C_v \approx 3.8 \quad (A2)$$

and

$$\frac{l_p}{Z} = C_l \quad C_l = 1.8 \quad (A3)$$

$$\frac{h_T}{Z} = C_h \quad \frac{C_h}{C_l} = 1.15 \quad (A4)$$

$$\frac{\Delta T}{\Delta T_m} = \exp[-(Z/l_p)^2]$$

$$\frac{w}{w_m} = \exp\left[-(r/l_v)^2\right].$$

Here,  $\Delta T_m$  and  $w_m$  are centerline temperature difference and velocity, and

$$Q^* = Q/(\rho_1 \sqrt{gZ} C_p T_1 Z^2) \quad (\text{A5})$$

is a dimensionless buoyancy parameter based on  $Q$ , which is the heat addition;  $\Delta\rho \equiv (\rho_1 - \rho)$  is positive, and  $l_v$  and  $l_t$  are velocity and temperature scale lengths.

Given these approximations, we can show that the mass averaged temperature and density in the plume are

$$\frac{\bar{\Delta T}}{T_1} = \frac{\bar{\Delta\rho}}{\rho_1} = \frac{1}{\pi C_v C_l^2} (Q^*)^{2/3} = \frac{Q}{\dot{m} C_p T_1} \quad (\text{A6})$$

and mass flow in the plume at a height  $Z$  is

$$\dot{m}_p = \rho_1 w_m \pi l_v^2 \quad (\text{A7})$$

or

$$\dot{m}_p = \rho_1 \sqrt{g} \frac{\bar{\Delta\rho}}{\rho_1} (\pi C_v C_l^2)^{3/2} (Z)^{5/2} \quad (\text{A8})$$

In (A8),  $(\bar{\Delta\rho}/\rho_1)$  was used to replace  $Q^*$  (which entered through  $w_m$  via Eqn (7b) by use of Eqn (A6) and  $\Delta T$  evaluated at  $Z$ ).



UNITED STATES DEPARTMENT OF COMMERCE  
National Bureau of Standards  
Washington, D.C. 20234

July 15, 1980

APPENDIX C

MEMORANDUM FOR Those Listed

From: Robert S. Levine, Chief  
Fire Science Division

Subject: A Preliminary Attempt to Assess a Material's  
Toxic Hazard from Toxicity Data

Any assessment of toxic hazard must be relative to some reasonable fire scenario. In this memo I have chosen as a scenario a fire in a small room the size of a bedroom with an open door. A fuel corresponding to wood is assumed to be burning at a steady, or very slowly increasing, rate. The target material, on a table 3 ft. above the floor, is heated by radiation from the hot gas layer in the upper part of the room, loses heat only by radiation, and so reaches an equilibrium temperature. It's decomposition products are mixed with the other gases (air and fuel products) leaving the room. For a given amount of the target material, the following "toxic hazard" statement is suggested.

$$\text{Toxic Hazard} = \frac{\dot{W}_T(\text{TF.})}{\dot{W}_f}$$

where:  $\dot{W}_T$  is the target material decomposition rate, gm/sec, at the temperature indicated

$\dot{W}_f$  is the fuel burn rate to create the given gas flow and gas temperature condition

(T.F.) is the toxicity factor, relative to wood, as measured by an agreed-upon protocol.

The following pages show the calculation. Briefly:

1st Assuming the room geometry, the mathematical fire Model RUNF is run at several Fire Sizes on the 7/32 computer. This calculates upper layer gas temperatures, flow rates, and layer depth.

2nd Assuming wood as a fuel, burning rates and gas compositions are estimated.

3rd The equilibrium temperature of the target material is estimated.

4th Given knowledge of the target material decomposition rate and toxicity relative to wood at the equilibrium temperature, toxic hazard is calculated. This will, of course, be a strong function of the assumed fire size. Alternately, a fire size can be calculated that will cause the material to reach a given temperature at which the toxicity data were obtained, and these data used to rate "Toxic Hazard".

Jim Quintiere's Program on 7/32 Computer -  
Steady State Fire in a Room

"RUNF"

Room 10' x 12' x 8' high =  $\overset{W}{3.05} \times \overset{L}{3.66} \times \overset{H}{2.44\text{m}}$  high

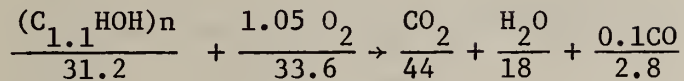
Doorway = 2' x 6' =  $\overset{H}{0.61} \times 1.83\text{m}$

Fire Size	Upper Gas Layer		Doorway Flow	
	Temp.	Height	cfm	
100 KW.	112°C-233F	0.4m (1.3 ft)	846 cfm - 0.48 Kg/sec	
300	246 474	0.95 1.5	1215	0.69
500	386 727	0.46 1.5	1269	0.72
600	473 845	0.46 1.5	1293	0.73
700	523 974	0.45 1.5	1292	0.73
800	595 1102	0.45 1.5	1285	0.73
900	666 1231	0.45 1.5	1276	0.72



# Calculate Fire Gases

Assume - Combustion of cellulose - wood.



Assume CO is 10% of CO<sub>2</sub> -----

[ Nike Site Mattress burns, CO = 0.1 to 0.2 CO<sub>2</sub>  
Bldg. 205 upholstered chair burns, CO = 0.05 to 0.1 CO<sub>2</sub> ]

heat of combustion = 12,000 Btu/lb = 6600 cal/gm.

assume 80% combustion efficiency = 5300 cal/gm.

Now calculate burning rate for "Fire Size - KW" values on pg. 1.

conversion: 11 gm-cal/sec = 4.187 watts

$$1 \text{ KW} = \frac{1000}{4.187} = 240 \text{ cal/sec} = .045 \frac{\text{gm fuel}}{\text{sec}}$$

Then:		Gases		Products		
Fire Size	Burn Rate	Mol/Sec	O <sub>2</sub> reqd.	CO	CO <sub>2</sub>	H <sub>2</sub> O
KW	gm/sec		mol/sec	mol/sec	mol/sec	mol/sec
100	4.5	0.144	0.151	0.0144	.144	.144
300	13.5	0.43	0.45	0.043	0.43	0.43
500	22.5	0.72	0.76	0.072	0.72	0.72
600	27.0	0.86	0.90	0.086	0.86	0.86
700	31.5	1.01	1.06	0.101	1.01	1.01
800	36.0	1.16	1.22	0.116	1.16	1.16
900	40.5	1.30	1.36	0.130	1.30	1.36

RUNF gives doorway flow in Kg/sec.

$$1 \text{ gm mole air} = 29 \text{ gms, } 1 \text{ Kg/sec} = \frac{1000}{29} = 34.5 \text{ gm moles/sec.}$$

$$O_2 \text{ concentration is 21\%. Molar Flow Rate} = (\text{Kg/sec}) (.21) (34.5) \\ = (7.2 \text{ moles/sec}) (\text{Kg/sec})$$

$$\text{From Page 1 (RUNF) } O_2 \text{ flow} = \frac{0.48 \times 7.2}{0.73 \times 7.2} = \frac{3.5}{5.3} \text{ moles/sec}$$

So we are not  $O_2$  limited in this series, since a max of 1.36 mol/sec is required.

---

Mol wt fuel  $1.1 \times 12 = 13.2$

31.2 gm	+ 16
	+ 2
	<hr/>
	31.2

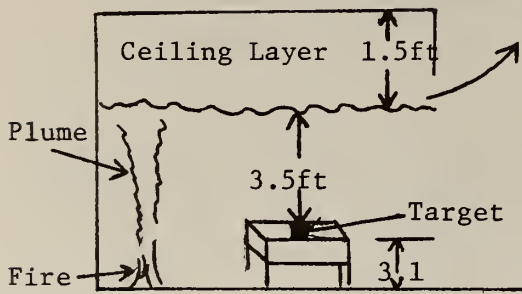
Calculate Upper Layer Gases, Contd.

Fire Size Kw	Air Flow Kg/sec, Moles/sec	1 N <sub>2</sub> Moles/sec	2 O <sub>2</sub> Moles/sec	3 O <sub>2</sub> Burned	4 O <sub>2</sub> Remain	5 CO	6 Moles/sec		7 H <sub>2</sub> O	8 Total Moles	Percent		Percent	
							CO <sub>2</sub>	H <sub>2</sub> O			CO	CO <sub>2</sub>	H <sub>2</sub> O	O <sub>2</sub>
100	0.48	15.8	12.5	0.151	3.35	0.0144	.144	.144	16.15	.087	.87	.87	.87	20.7
300	0.69	23.9	18.8	0.45	4.45	0.043	.43	.43	24.15	.178	1.78	1.78	1.78	18.4
500	0.72	24.9	19.6	0.76	4.44	0.072	.72	.72	25.55	.282	2.82	2.82	2.82	17.38
600	0.73	25.1	19.9	0.90	4.4	0.086	.86	.86	26.11	.329	3.29	3.29	3.29	16.85
700	0.73	25.1	19.9	1.06	4.24	0.101	1.01	1.01	26.26	.384	3.84	3.84	3.84	16.14
800	0.73	25.1	19.9	1.22	4.08	0.116	1.16	1.16	26.42	.439	4.39	4.39	4.39	15.44
900	0.72	24.9	19.6	1.36	3.84	0.130	1.30	1.30	26.17	.497	4.97	4.97	4.97	14.67

Table 2 - Gas Flow Rates and Composition

Column 8 = Columns 1 + 4 + 5 + 6 + 7

Calculate equilibrium temperature of object at 3 ft level in the center of



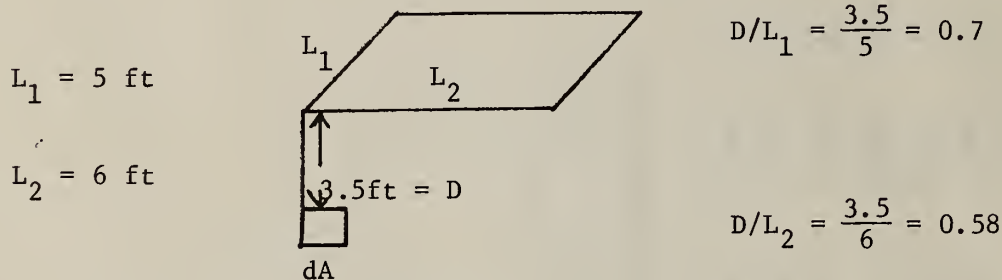
the room. - Ceiling gas of temperature and composition previously calculated, object is insulated - loses heat by radiation only -

$$\epsilon_T = 1.0.$$

$$\sigma \left[ \epsilon_g F_{1-2} T_g^4 + (1-\epsilon_g) F_{1-2} T_w^4 \right] = \sigma \epsilon_T F_{2-3} (T_T)^4$$

The view factor  $F_{1-2}$  for the target receiving radiation from the hot gas can be evaluated from Hottel<sup>(1)</sup> Fig. 4.

Assume the target is the area,  $dA$ , and sees four rectangles as shown here: (1/4 the room area)



$$D/L_1 = \frac{3.5}{5} = 0.7$$

$$D/L_2 = \frac{3.5}{6} = 0.58$$

$F = 0.19$  for each of the four rectangles

$$\text{then } F_{1-2} = 4F = \underline{.76}$$

$F_{2-3}$ , the view factor for radiation from the target is assumed to be 1.0.

$\epsilon_g$  can be calculated from the gas composition already calculated, plus a soot correction. The higher  $\epsilon_g$ , the larger the 1st term and the smaller the 2nd term in the equation above. If we take a worst case and assume the wall temperature,  $T_w$ , reaches the gas temperature  $T_g$ , this is equivalent to gas radiation alone with  $\epsilon_g = 1.0$ . Making this assumption:

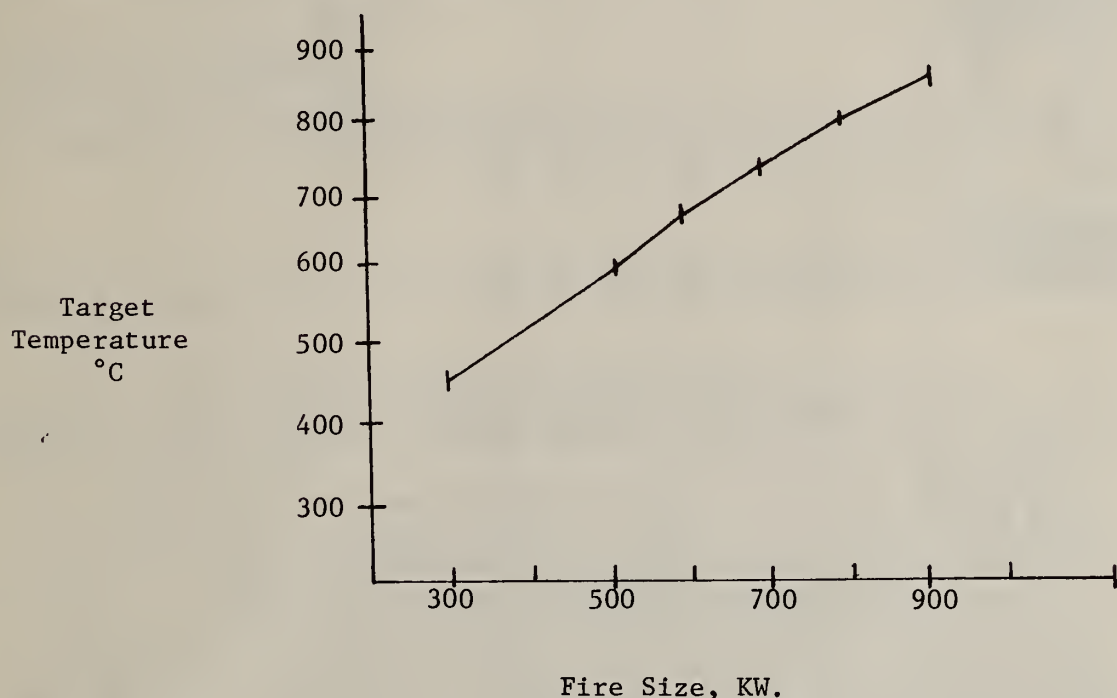
$$F_{1-2} T_g^4 = T_T^4, \text{ or } T_T = T_g \sqrt[4]{F_{1-2} \frac{\epsilon_g}{\epsilon_T}} = T_g (0.76)^{1/4}$$

$$T_T = 0.92 T_g \quad (\text{both temperatures in } ^\circ\text{K})$$



- (1) McAdams, heat transmission - 3rd Ed (1954) pg. 68. Chapter written by H. C. Hottel.

If we want to evaluate the toxic hazard at a particular temperature as used in the protocol, then plot  $T_T$  vs. fire size in KW--and evaluate the flow rates, material and gas composition at this protocol temperature.



This Scenario - Plotted from Table 3

### Summary

With this calculation, the contribution of the target material to the toxic hazard can be judged from the next table (where  $\dot{W}_g$  is the decomposition rate, gm/sec, of the target material at temp  $T_T$ ). Obviously,  $\dot{W}_g$  and the "Toxic Factor" must be obtained by some means outside the scope of this calculation.

Summary of Calculated Data  
10' x 12' x 8' Room with 2' x 6' Doorway

Fire Size KW	$\dot{W}_f$ Fuel Burn Rate gm/sec	Upper Gas Layer					Target Material		
		Temperature °C		Percent O <sub>2</sub>		Flow Rate gm/sec		Temp. °C	$\dot{W}_T$ Decomp Rate gm/sec
		°C	°K	CO	CO <sub>2</sub>	CO	O <sub>2</sub>	°K	
100	4.5	112	385	0.087	0.87	0.4	6.3	81	354
300	13.5	246	519	0.178	1.78	1.2	19	203	476
500	22.5	386	659	0.282	2.82	2.01	31.7	334	607
600	27.0	473	746	0.329	3.29	2.4	38	414	687
700	31.5	523	796	0.384	3.84	2.8	44	459	732
800	36.0	595	868	0.439	4.39	3.2	51	526	799
900	40.5	666	939	0.497	4.97	3.65	57	590	863

Table 3

$$\text{Toxic Hazard} = \frac{\dot{W}_T (\text{Toxic Factor})}{\dot{W}_f}$$

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4. TITLE AND SUBTITLE  Mathematical Modeling of Fires		5. Publication Date  September 1980	
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7. AUTHOR(S)  Robert S. Levine		8. Performing Organ. Report No.	
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		14. Sponsoring Agency Code	
15. SUPPLEMENTARY NOTES  <input type="checkbox"/> Document describes a computer program; SF-185, FIPS Software Summary, is attached.			
16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)  <p>This presentation has three technical parts, and ends with audience participation and recommendations. First, a brief discussion of fire growth in a compartment is presented, showing why we need full scale tests, or a mathematical model adequately simulating such growth. The second part of the talk describes what several Federal agencies and their grantees are doing to bring about the necessary engineering and mathematical capability for this modeling. The third part illustrates some problems that may be of interest to fire protection engineers that can be solved relatively simply by using fragments of the modeling capability now available.</p> <p>Then a discussion was held with the audience to determine modeling needs. Should we provide a series of simple models, each applicable to a limited range of problems, or a major comprehensive model, accessible from a computer terminal, that will solve a very wide range of problems? The audience decided both were needed.</p>			
17. KEY WORDS (six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)  Fire; fire engineering; fire safety; mathematical modeling; modeling application.			
18. AVAILABILITY  <input checked="" type="checkbox"/> Unlimited  <input type="checkbox"/> For Official Distribution. Do Not Release to NTIS  <input type="checkbox"/> Order From Sup. of Doc., U.S. Government Printing Office, Washington, DC 20402, SD Stock No. SN003-003-  <input checked="" type="checkbox"/> Order From National Technical Information Service (NTIS), Springfield, VA. 22161		19. SECURITY CLASS (THIS REPORT)  UNCLASSIFIED  20. SECURITY CLASS (THIS PAGE)  UNCLASSIFIED	21. NO. OF PRINTED PAGES  44  22. Price  \$6.00







